Loop Quantum Cosmology: Relation Between Theory and Observations 90

圈量子宇宙学: 理论与观测的关系 90

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Contents

目录

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Abstract

摘要

This chapter provides a review of the frameworks developed for cosmological perturbation theory in loop quantum cosmology and applications to various models of the early universe including inflation, ekpyrosis, and the matter bounce, with an emphasis on potential observational consequences. It also includes a discussion on extensions to include non-Gaussianities and background anisotropies, as well as on its limitations concerning trans-Planckian perturbations and quantization ambiguities. It concludes with a summary of recent work studying the relation between loop quantum cosmology and full loop quantum gravity.

本章综述了圈量子宇宙学中针对宇宙学微扰理论建立的框架，及其在包括暴胀、火劫宇宙论和物质反弹在内的多种早期宇宙模型中的应用，重点关注了潜在的可观测效应。本章还讨论了该框架对非高斯性和背景各向异性的扩展，以及它在跨普朗克微扰与量子化歧义方面存在的局限性。最后总结了近期研究圈量子宇宙学与完整圈量子引力之间关系的成果。

Keywords

关键词

Loop quantum gravity . Loop quantum cosmology - Cosmological perturbation theory - Cosmic microwave background - Inflation

圈量子引力。圈量子宇宙学——宇宙扰动理论——宇宙微波背景——暴胀

Introduction

引言

Loop quantum cosmology (LQC) is a quantum theory for the gravitational field of homogeneous spacetimes commonly used in cosmology that is based on the techniques of loop quantum gravity (LQG). As has been reviewed in the previous chapter of this book, in recent years LQC has led to significant insights and detailed results concerning the quantization of these cosmological models, and fundamental questions have been addressed-in particular, the classical big bang singularity is resolved by a non-singular bounce due to quantum gravity effects. LQC thus provides a detailed description of the spacetime geometry during the Planck era for Friedman-Lemaître-Robertson-Walker (FLRW) and Bianchi spacetimes.

圈量子宇宙学 (LQC) 是基于圈量子引力 (LQG) 方法建立的、适用于宇宙学中均匀时空的引力场量子理论。正如本书前一章所述，近年来 LQC 在这些宇宙学模型的量子化问题上取得了重要洞见与详尽结果，解决了多个基础问题——尤其是经典大爆炸奇点因量子引力效应被非奇异反弹取代。因此，LQC 可以详细描述弗里德曼-勒梅特-罗伯逊-沃尔克 (FLRW) 时空与比安基时空在普朗克纪元的时空几何。

The natural next step is to use LQC to extend our current model of the early universe to include Planck scale physics, by developing a framework for cosmological perturbation theory in LQC. The goal of such an extension is twofold. On the one hand, it would allow us to overcome the limitations of general relativity, on which the standard cosmological model rests, and to achieve a more complete picture of the past history of the cosmos. And, on the other hand, such an extension could potentially connect Planck scale physics with observations-in particular of the cosmic microwave background (CMB), the faint afterglow of the primordial universe-thereby opening an avenue to test some of the ideas on which this approach to quantum gravity rests. Describing the state of the art of this research program in a pedagogical yet comprehensive way is the primary goal of this chapter.

下一步自然是在 LQC 中建立宇宙扰动理论框架，将我们的早期宇宙模型拓展至包含普朗克尺度物理。这一拓展有双重目标: 一方面，它能帮助我们突破标准宇宙模型所依托的广义相对论的局限，得到更完整的宇宙过去演化图景；另一方面，这一拓展有望将普朗克尺度物理与观测联系起来——尤其是对宇宙微波背景 (CMB，原初宇宙残留的微弱辐射) 的观测，从而开辟一条检验该量子引力方法核心思想的途径。以教学式的清晰方式全面介绍该研究方向的最新进展，是本章的核心目标。

An additional motivation to study cosmological perturbations is more conceptual. Homogeneous spacetimes have a finite number of degrees of freedom, while cosmological perturbations are described by fields with local degrees of freedom. Cosmology, in addition to offering the possibility of comparing predictions to observations, also offers a simple testing ground for various tools and techniques of full quantum gravity, and this testing ground will be vastly enriched by extending it to have local degrees of freedom.

研究宇宙扰动的另一项动机偏概念性: 均匀时空只有有限个自由度，而宇宙扰动由具有局域自由度的场描述。宇宙学除了能让我们将预言和观测对比，还为完整量子引力的各类工具和方法提供了简单的测试场地，将该场地拓展至包含局域自由度后，其内容会极大丰富。

Including local degrees of freedom is a challenging task, as the simplifying consequences of exact homogeneity can no longer be used. A variety of different approaches to extend LQC to include cosmological perturbations have been developed during the last decade, each with some simplifying assumptions and some strengths and weaknesses.

引入局域自由度是一项极具挑战性的工作，因为我们无法再利用严格均匀性带来的简化便利。过去十年间，研究者已经开发了多种不同的方法将 LQC 拓展至包含宇宙扰动，每种方法都做了一定的简化假设，也各有优劣。

Importantly, these various extensions of LQC can provide a quantum gravity extension to many types of cosmological models including inflation, ekpyrosis, and the matter bounce. For example, in the standard inflationary scenario, one normally starts the evolution far from the Planck era, when the curvature and energy density of matter fields in the universe is around twelve orders of magnitude below the Planck scale and quantum gravity effects are negligible. Our ignorance about the earlier stages of cosmic evolution is encoded in the choice of initial conditions at the onset of inflation, for both the background homogeneous geometry and cosmological perturbations, with the latter typically assumed to be in the so-called Bunch-Davies vacuum at the onset of inflation. This is a key, yet strong assumption. It is of considerable interest to extend this scenario backwards in time to include the Planck era and to show that such initial conditions (or something close to them) can be derived as a result of the pre-inflationary dynamics when quantum gravity effects become important. Further, in typical inflationary models, the background spacetime is classically singular, but this is cured in LQC where quantum gravity effects replace the big bang singularity by a non-singular bounce. Similarly, both the ekpyrotic and matter bounce scenarios require a cosmic bounce, and LQC provides a natural mechanism for such bounce.

值得注意的是，这些不同的 LQC 拓展可以为多种宇宙模型提供量子引力延伸，包括暴胀模型、火劫模型和物质反弹模型。例如，在标准暴胀场景中，演化通常起始于远离普朗克纪元的时刻，此时宇宙的曲率和物质场能量密度比普朗克尺度低 12 个数量级，量子引力效应可以忽略。我们对宇宙更早演化阶段的无知体现在暴胀起始阶段初始条件的选择中，这既针对背景均匀几何，也针对宇宙扰动——通常假设扰动在暴胀开始时处于所谓的邦奇-戴维斯真空。这是一个关键但很强的假设。将该场景向后延伸至普朗克纪元，证明当量子引力效应变得重要时，这类初始条件 (或近似这类的初始条件) 可以从暴胀前的动力学中推导出来，是相当有研究价值的。此外，在典型的暴胀模型中，背景时空在经典上是奇异的，但这一问题在 LQC 中得到解决: 量子引力效应用非奇异反弹取代了大爆炸奇点。类似地，火劫模型和物质反弹模型都需要宇宙反弹，而 LQC 恰好为这类反弹提供了自然机制。

This chapter provides an overview of how LQC provides a quantum gravity completion of these cosmological scenarios and highlights the extra features that this extension adds to observable quantities. These new effects, due to quantum gravity, are a window to the Planck era of the cosmos and can be used to test the ideas discussed here by comparing the predictions to observations of the CMB. In addition, these results also provide insight on how standard quantum field theory can emerge from a background-independent approach to quantum gravity, among other foundational questions for quantum cosmology.

本章概述了 LQC 如何为这些宇宙场景提供量子引力完备化，并强调了这一拓展给可观测物理量带来的额外特征。这些由量子引力产生的新效应是通往宇宙普朗克纪元的窗口，可以通过将预言与 CMB 观测对比，来检验本文讨论的观点。此外，这些结果还在量子宇宙学的基础问题上，例如标准量子场论如何从背景独立的量子引力方法中演生出来，提供了新的洞见。

The outline of this chapter is the following: the section ”Standard Cosmology: Review and Guidance for Quantum Gravity” provides a brief review of standard cosmological perturbations. Then the section ”Cosmological Perturbations in LQC” presents four different and complementary LQC-based frameworks for cosmological perturbation theory, and the section ”Predictions for the CMB” describes the results of applying these frameworks to different cosmological models, including inflation and some alternatives to inflation. Extensions to include non-Gaussianities and background anisotropies are reviewed in the section ”Extensions”, and some limitations are discussed in the section ”Limitations”. Finally, we end with some comments on the link between LQC and LQG in the section ”Beyond LQC”. We use natural units where $c = G = \hbar = 1$.

本章大纲如下:“标准宇宙学: 对量子引力的回顾与启发”一节简要回顾了标准宇宙学微扰理论。随后“圈量子宇宙学中的宇宙学微扰”一节介绍了四种基于圈量子宇宙学、彼此互补的不同宇宙学微扰理论框架,“宇宙微波背景的预言”一节阐述了将这些框架应用于不同宇宙学模型 (包括暴胀模型与若干非暴胀替代模型) 得到的结果。“拓展内容”一节回顾了引入非高斯性与背景各向异性的扩展研究，“局限性”一节探讨了该领域现存的一些局限。最后，我们在“圈量子宇宙学之外”一节对圈量子宇宙学与圈量子引力的关联做了总结讨论。本文采用自然单位制，其中 $c = G = \hbar = 1$ 。

Standard Cosmology: Review and Guidance for Quantum Gravity

标准宇宙学: 量子引力的综述与指引

Before discussing quantum gravity effects on cosmological perturbations, it is useful to review standard cosmological perturbation theory based on quantum field theory on a fixed classical background; for a detailed introduction, see, e.g., [167]. In addition to setting the notation and pointing out some key results, this discussion will give some basic intuition about the dynamics of cosmological perturbations and provide some hints as to where quantum gravity effects may be expected to arise-for pedagogical purposes, this last part of the discussion is qualitative in this section; the way this picture concretely emerges in LQC is presented later in this chapter.

在讨论量子引力对宇宙学微扰的影响之前，先回顾基于固定经典背景上量子场论的标准宇宙学微扰论是很有必要的；详细介绍参见例如文献 [167]。除了确定记号、指出部分关键结论外，本讨论还将帮助读者建立关于宇宙学微扰动力学的基本直觉，并提示量子引力效应可能在哪些地方出现——出于教学目的，本部分讨论在本节仅作定性分析；该图景在圈量子宇宙学中如何具体呈现，将在本章后续介绍。

Throughout this chapter, we mostly focus on scalar perturbations to shorten the discussion. We provide references to the relevant literature for the interested reader for details on tensor and vector modes.

为简化讨论，本章通篇我们主要聚焦标量微扰。对于感兴趣想要了解张量模和矢量模细节的读者，我们在相关文献中提供了参考资料。

For concreteness, we will consider cosmological perturbations on a spatially flat FLRW background geometry for the case that the matter content is a minimally coupled scalar field ϕ sourced by a potential $V(\phi)$. (Other matter fields and homogeneous background geometries are possible; see, e.g., the subsection ”Anisotropies” for a summary on the extension to Bianchi I). Perturbations to the metric tensor and the scalar field, $\delta g_{ab}(\mathbf{x}, t)$ and $\delta\phi(\mathbf{x}, t)$, contain three physical degrees of freedom: a scalar mode due to matter and two tensor modes of purely gravitational origin, corresponding to the two polarizations of gravitational waves. Vector perturbations can be ignored in this scenario, as they do not get excited by scalar matter.

具体而言，我们考虑空间平坦 FLRW 背景几何上的宇宙学微扰，其中物质内容是由势 $V(\phi)$ 源发的最小耦合标量场 ϕ 。(也可以考虑其他物质场和齐性背景几何；例如关于扩展到 Bianchi I 型的总结参见“各向异性”小节)。度规张量和标量场的微扰 $\delta g_{ab}(\mathbf{x}, t)$ 和 $\delta\phi(\mathbf{x}, t)$ 包含三个物理自由度: 一个来自物质的标量模，以及两个纯引力起源的张量模，对应引力波的两种偏振。矢量微扰在该情形下可以忽略，因为它们不会被标量物质激发。

The scalar mode can be described by the field $\mathcal{Q}$ that is gauge-invariant in the sense that it is invariant under linear diffeomorphisms; note that $\mathcal{Q}$ is related to the familiar comoving curvature perturbation $\mathcal{R}$ and the Mukhanov-Sasaki variable v by $\mathcal{Q} \equiv z/a\mathcal{R} = v/a$, where $z = a\dot{\phi}/H$ and, as usual, $a(t)$ is the scale factor of the background spacetime, $H(t) = \dot{a}/a$ is the Hubble rate, and a dot denotes a derivative with respect to the cosmic time.

该标量模可以用规范不变场 $\mathcal{Q}$ 描述，其规范不变性体现在它在线性微分同胚变换下保持不变；注意 $\mathcal{Q}$ 和我们熟悉的共动曲率扰动 $\mathcal{R}$ 、Mukhanov-Sasaki 变量 v 满足关系 $\mathcal{Q} \equiv z/a\mathcal{R} = v/a$ ，其中 $z = a\dot{\phi}/H$ ，按惯例 $a(t)$ 是背景时空的尺度因子，$H(t) = \dot{a}/a$ 是哈勃率，点号表示对宇宙时间求导。

The classical phase space of interest is therefore $\Gamma_{\text{phys}} = \Gamma_{\text{hom}} \otimes \Gamma_{\text{pert}}$, where Γ_{hom} is the standard phase space of FLRW geometry, made of the two canonical pairs $\left(a, \pi_a, \phi, p_\phi\right)$, and Γ_{pert} is the phase space of scalar and tensor perturbations. Γ_{pert} is commonly called the ”reduced phase space” of the perturbations because it only includes the gauge-invariant degrees of freedom of $\delta g_{ab}(\mathbf{x}, t)$ and $\delta\phi(\mathbf{x}, t)$.

因此我们关心的经典相空间为 $\Gamma_{\text{phys}} = \Gamma_{\text{hom}} \otimes \Gamma_{\text{pert}}$ ，其中 Γ_{hom} 是 FLRW 几何的标准相空间，由两个正则对 (a, π_a, ϕ, p_ϕ) 构成，Γ_{pert} 是标量和张量微扰的相空间。Γ_{pert} 通常被称为微扰的“约化相空间”，因为它仅包含 $\delta g_{ab}(\mathbf{x}, t)$ 和 $\delta\phi(\mathbf{x}, t)$ 的规范不变自由度。

Still in the classical theory, the dynamics is determined as follows. The background dynamics in Γ_{hom} is simply determined by the usual Friedman equations for FLRW spacetimes, while the dynamics of the scalar is dictated by the true Hamiltonian (in Fourier space)

即使在经典理论中，动力学也按如下方式确定: Γ_{hom} 中的背景动力学简单由 FLRW 时空的常规弗里德曼方程决定，而标量的动力学由 (傅里叶空间中的) 真实哈密顿量支配

$$\mathcal{C}_2^{(\mathcal{Q})}[N] = \frac{N}{2(2\pi)^3} \int d^3k \left(\frac{1}{a^3} \left| \mathfrak{p}_{\mathbf{k}}^{(\mathcal{Q})} \right|^2 + a \left(k^2 + \mathcal{U} \right) |\mathcal{Q}_{\mathbf{k}}|^2 \right), \tag{1}$$

where N is the lapse function. This expression is obtained by expanding the scalar constraint of general relativity up to second order. Note that the scalar mode behaves as a minimally coupled scalar field with an effective potential

其中 N 是移距函数。该表达式是将广义相对论的标量约束展开到二阶得到的。可以看到，该标量模的行为类似于带有效势的最小耦合标量场

$$\mathcal{U} = a^2 \left[V(\phi) r - 2V_\phi(\phi)\sqrt{r} + V_{\phi\phi}(\phi) \right], \tag{2}$$

where $V_\phi(\phi)$ and $V_{\phi\phi}(\phi)$ are the first and second derivatives of the scalar field potential $V(\phi)$ with respect to ϕ , while $r \equiv 3\kappa p_\phi^2 / \left[p_\phi^2/2 + a^6 V(\phi) \right]$. The equation of motion, expressed in terms of conformal time η , is

其中 $V_\phi(\phi)$ 和 $V_{\phi\phi}(\phi)$ 分别是标量场势 $V(\phi)$ 对 ϕ 的一阶和二阶导数, 而 $r \equiv 3\kappa p_\phi^2 / \left[p_\phi^2/2 + a^6 V(\phi) \right]$ 。用共形时间 η 表示的运动方程为

$$\mathcal{D}_{\mathbf{k}}'' + 2\frac{a'}{a}\mathcal{Q}_{\mathbf{k}}' + \left(k^2 + \mathcal{U}(\eta) \right) \mathcal{Q}_{\mathbf{k}} = 0. \tag{3}$$

Here primes denote derivatives with respect to the conformal time, with $f' = a\dot{f}$. This is a second-order ordinary differential equation with coefficients given by the background quantities, such as the scale factor $a(\eta)$ and its derivative. The dynamics for tensor modes are very similar, except with no effective potential, $\mathcal{U}_{\text{tensor}} = 0$.

此处撇号表示对共形时间的导数，满足 $f' = a\dot{f}$ 。这是一个二阶常微分方程，系数由背景量给出，例如标度因子 $a(\eta)$ 及其导数。张量模式的动力学性质十分相似，区别仅在于张量模式没有有效势，即 $\mathcal{U}_{\text{tensor}} = 0$ 。

It can be convenient to rewrite the equation of motion in terms of other variables describing the scalar mode, for example, for the Mukhanov-Sasaki variable $v = aQ$

将运动方程改用其他描述标量模式的变量改写会更方便，例如，针对穆哈诺夫-佐佐木变量 $v = aQ$

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0, \tag{4}$$

and $z''/z = a''/a - \mathcal{U}$.

且有 $z''/z = a''/a - \mathcal{U}$ 。

Therefore, in this perturbative approach, one first solves the evolution for the background degrees of freedom $a(\eta)$ and $\phi(\eta)$, while ignoring perturbations; the solution fixes the background spacetime metric once and for all. Next, the solution for $a(\eta)$ and $\phi(\eta)$ is needed for Eq. (3) whose solution, in turn, determines the dynamics of the linear perturbations. This completes the summary of the classical theory.

因此，在该微扰方法中，我们首先忽略微扰，求解背景自由度 $a(\eta)$ 和 $\phi(\eta)$ 的演化；该解会一次性确定背景时空度规。接下来，方程 (3) 需要用到 $a(\eta)$ 和 $\phi(\eta)$ 的解，而方程 (3) 的解又会确定线性微扰的动力学。至此经典理论的概述就完成了。

In approaches to the early universe such as inflation, one further proceeds to quantize the perturbations while leaving the background geometry classical. The homogeneous phase space Γ_{hom} and its dynamics are unmodified, but the classical phase space for the perturbations Γ_{pert} is replaced by a Hilbert space $\mathcal{H}_{\text{pert}}$, and the quantum dynamics is determined by interpreting (3) as the Heisenberg equation for the operator $\hat{\mathcal{Q}}_{\mathbf{k}}$.

在暴胀这类研究早期宇宙的方法中，研究者会进一步将微扰量子化，同时保留背景几何的经典属性。齐相位空间 Γ_{hom} 及其动力学保持不变，但微扰的经典相空间 Γ_{pert} 会被替换为希尔伯特空间 $\mathcal{H}_{\text{pert}}$ ，将式 (3) 诠释为算符 $\hat{\mathcal{Q}}_{\mathbf{k}}$ 的海森堡方程即可确定量子动力学。

Importantly, z''/z introduces a scale into the dynamics as can be most clearly seen from Eq. (4). In inflation, this length scale is the Hubble radius $r_H = 1/H$, but more generally $\sqrt{|z/z''|}$ gives a measure of the radius of curvature of the background spacetime. Given the importance of this lengthscale, it is natural to split the Fourier modes v_k into two categories: those with a wavelength shorter than $\sqrt{|z/z''|}$ and those with a longer wavelength.

重要的是，z''/z 给动力学引入了一个特征标度，这一点从式 (4) 中可以看得最清楚。在暴胀中，这个长度标度是哈勃半径 $r_H = 1/H$ ，但更一般地，$\sqrt{|z/z''|}$ 可以衡量背景时空的曲率半径。鉴于该长度标度的重要性，我们很自然地将傅里叶模式 v_k 分为两类: 波长小于 $\sqrt{|z/z''|}$ 的模式，和波长大于 $\sqrt{|z/z''|}$ 的模式。

For short-wavelength modes $\left(k^2 \gg |z''/z|\right)$, the term $|z''/z|$ in (4) is unimportant; in plain words, short-wavelength modes do not "feel" the curvature of the background spacetime and evolve exactly as in the Minkowski space, while long-wavelength perturbations $\left(k^2 \ll |z''/z|\right)$ do feel the spacetime curvature and evolve differently, with in this case the k^2 term in the equation of motion being negligible.

对于短波长模式 $\left(k^2 \gg |z''/z|\right)$，式 (4) 中的 $|z''/z|$ 项并不重要；简而言之，短波长模式不会“感知”到背景时空的曲率，其演化和闵可夫斯基空间中的演化完全一致，而长波长微扰 $\left(k^2 \ll |z''/z|\right)$ 会感知到时空曲率，演化规律也不同，在此情况下运动方程中的 k^2 项可以忽略。

This simple fact of the physics underlying cosmological perturbations helps to understand where quantum gravity effects might arise. There are two obvious possible regimes: (i) for short-wavelength modes that have a wavelength comparable to the Planck length $\ell_{\rm Pl}$ and (ii) for long-wavelength modes at a time when the dynamics of the background spacetime are modified by quantum gravity effects (since this will in turn introduce corrections in z''/z which impacts long-wavelength perturbations).

宇宙微扰背后的这一简单物理规律有助于我们理解量子引力效应可能在何处出现。有两种明显的可能区域:(i) 波长与普朗克长度相当的短波长模式 $\ell_{\rm Pl}$，(ii) 背景时空动力学已经被量子引力效应修改的时期的长波长模式 (因为这会反过来给 z''/z 引入修正，进而影响长波长微扰)。

For the first possibility, modifications to the equations of motion for cosmological perturbations may arise for wavelengths $\lambda \sim \ell_{\rm Pl}$. Recalling that short-wavelength modes evolve just as in the Minkowski space, quantum gravity effects for these modes are typically modeled as modifications to the dispersion relation for the perturbations with the equation of motion becoming $v_k'' + f(k)^2 v_k = 0$ where $f(k)$ depends only on k and is independent of the scale factor $a(t)$ and other geometric degrees of freedom of the background spacetime and $f(k) \rightarrow k$ in the limit $\lambda \gg \ell_{\rm Pl}$. It has been shown that so long as $f(k)$ is real and the background dynamics is sufficiently smooth so the quantum state $|v_k\rangle$ evolves adiabatically, modified dispersion relations will not have any impact on predictions for the CMB [69].

对于第一种可能性，当波长为 $\lambda \sim \ell_{\rm Pl}$ 时，宇宙学扰动的运动方程可能会发生修改。回顾可知，短波长模式的演化与闵氏空间中一致，这类模式的量子引力效应通常被建模为对扰动色散关系的修正，修正后的运动方程变为 $v_k'' + f(k)^2 v_k = 0$，其中 $f(k)$ 仅依赖于 k，与尺度因子 $a(t)$ 及背景时空的其他几何自由度无关，且在 $\lambda \gg \ell_{\rm Pl}$ 极限下满足 $f(k) \rightarrow k$。研究表明，只要 $f(k)$ 为实数，且背景动力学足够光滑使得量子态 $|v_k\rangle$ 绝热演化，修正后的色散关系就不会对 CMB 的预言产生任何影响 [69]。

This leaves the second possibility: the evolution of long-wavelength perturbations depends on the background spacetime, and if the dynamics of the background are modified due to quantum gravity effects, then these will in turn have an impact on long-wavelength perturbations, which can potentially show up in the CMB.

这就留下了第二种可能性: 长波长扰动的演化依赖于背景时空，若背景动力学因量子引力效应发生修改，那么修改反过来会影响长波长扰动，该效应有可能在 CMB 中显现。

This expectation due to general heuristic arguments is in fact realized and made precise by ab initio calculations in LQC, as shall be reviewed below.

这种由广义启发式论证得到的猜想，实际上已经通过圈量子宇宙学 (LQC) 的从头计算得到了验证和精确化，我们将在下文回顾相关内容。

Cosmological Perturbations in LQC

圈量子宇宙学中的宇宙微扰

Progress in understanding the very early universe has been possible due to a key observation of the CMB: the early universe was extraordinarily homogeneous and isotropic [23], and departures from exact homogeneity are sufficiently small that they can be described by linear perturbation theory on a FLRW geometry.

对极早期宇宙的认知进展得益于宇宙微波背景的一项关键观测: 早期宇宙极为均匀且各向同性 [23], 与完全均匀的偏离足够小，因此可以通过 FLRW 几何上的线性微扰理论描述。

As reviewed in the section ”Standard Cosmology: Review and Guidance for Quantum Gravity”, in standard cosmology the dynamics of the background and the perturbations are derived from the Einstein equations; the background is treated classically, while the fields describing the perturbations are quantized using linear quantum field theory in curved spacetimes, neglecting backreaction on the background spacetime.

正如“标准宇宙学: 量子引力的回顾与指引”一节所述，在标准宇宙学中，背景与微扰的动力学都由爱因斯坦方程推导得出；背景按经典处理，描述微扰的场则利用弯曲时空下的线性量子场论量子化，忽略微扰对背景时空的反作用。

It is not possible to follow the same procedure in LQG because the quantum analog of the Einstein equations is not yet fully understood. Much of the work in LQC initially focused on homogeneous spacetimes, but now there has been considerable work to extend the formalism of LQC to include perturbations. Here we summarize several strategies that have been developed to study cosmological perturbations in LQC, emphasizing their strengths and weaknesses.

由于爱因斯坦方程的量子对应尚未被完全理解，圈量子引力中无法沿用这套流程。圈量子宇宙学的早期研究大多聚焦均匀时空，目前已有大量工作将圈量子宇宙学的形式体系拓展至包含微扰的情形。本文总结了目前已发展出的几种研究圈量子宇宙学中宇宙微扰的策略，重点说明它们的优势与不足。

Dressed Metric Approach

修饰度规方法

The dressed metric approach to cosmological perturbation theory in LQC is based on a framework for quantum field theory on a quantum background spacetime [32]. It follows the strategy of semi-classical cosmology: restrict attention, already in the classical theory, to FLRW geometries with linear, gauge-invariant perturbations, and then quantize this sector [9, 10] . This is a natural extension of LQC applied to homogeneous spacetimes, where homogeneity is imposed classically and it is only the phase space reduced to the homogeneous degrees of freedom that is quantized. The idea underlying the dressed metric approach is to now include linear perturbations in the phase space that is to be quantized.

圈量子宇宙学 (LQC) 中宇宙扰动理论的修饰度规方法,基于量子背景时空上的量子场论框架 [32]。它遵循半经典宇宙学的策略: 早在经典理论阶段就将研究范围限定为带有线性规范不变扰动的 FLRW 几何，随后对该 sector 量子化 [9, 10] 。这是应用于齐性时空的 LQC 的自然延伸，在原 LQC 中齐性是在经典层面施加的，只有约化为齐性自由度的相空间才被量子化。修饰度规方法的核心思想是，现在要在线性扰动纳入待量子化的相空间中。

This approach rests on three key assumptions. (i) Gauge-invariant perturbations are defined at the classical level; this assumes that quantum corrections to the definition of gauge-invariant perturbations are subleading compared to other quantum gravity effects. (ii) The backreaction of perturbations on the homogeneous geometry is neglected, as is standard for linear perturbation theory; the consistency of this assumption is checked a posteriori by verifying that perturbations remain sufficiently small, as is also done in the semiclassical theory. (iii) The quantization is based on a hybrid approach, where the homogeneous degrees of freedom are quantized following LQC, while perturbations are handled by standard quantum field theory. This hybrid approach was first introduced for Gowdy cosmologies [159] and ignores the effects of potential LQC corrections to the equations of motion for the perturbations themselves, which is motivated by the fact that the energy-momentum of curvature perturbations always remains well below the Planck scale [11].

该方法基于三个关键假设。(i) 规范不变扰动在经典层面定义；该假设假设，规范不变扰动定义的量子修正与其他量子引力效应相比是次阶的。(ii) 与线性微扰理论的标准处理一致，忽略微扰对齐性几何的反作用；和半经典理论中的操作一样，该假设的自洽性会通过验证微扰始终足够小来事后检验。(iii) 量子化基于混合方法: 齐性自由度按照 LQC 规则量子化，而微扰则由标准量子场论处理。这种混合方法最初是为 Gowdy 宇宙学引入的 [159]，它忽略了潜在 LQC 对微扰自身运动方程的修正，该处理的依据是曲率扰动的能量动量始终远低于普朗克能标 [11]。

In general terms, this approach follows a very similar strategy to the strategy reviewed in the section "Standard Cosmology: Review and Guidance for Quantum Gravity", with the important difference that the background geometry is now also quantized: the classical phase space Γ_{hom} is replaced by the Hilbert space $\mathcal{H}_{LQC}$ of LQC for FLRW spacetimes, described in the Handbook's previous chapter on homogeneous LQC. The FLRW background is therefore no longer described by classical functions $a(\eta)$ and $\phi(\eta)$, but instead by a wave function $\Psi_{\text{hom}}(a, \phi)$. The challenge now is to determine the dynamics for the perturbations given the quantum background spacetime $\Psi_{\text{hom}}(a, \phi)$.

总体而言，该方法遵循的策略与“标准宇宙学: 量子引力的回顾与指引”一节回顾的策略非常相似，一个重要区别是此处背景几何也被量子化了: 经典相空间 Γ_{hom} 被替换为 FLRW 时空 LQC 的希尔伯特空间 $\mathcal{H}_{LQC}$，该空间在本手册此前关于齐性 LQC 的章节中已有介绍。因此 FLRW 背景不再由经典函数 $a(\eta)$ 和 $\phi(\eta)$ 描述，而是由波函数 $\Psi_{\text{hom}}(a, \phi)$ 描述。当前的挑战是，在给定量子背景时空 $\Psi_{\text{hom}}(a, \phi)$ 的情况下确定微扰的动力学。

In more detail, following the principles of LQG, the dynamics for perturbations in the dressed metric approach is derived starting from the Wheeler-de Witt equation $\hat{H}_{\text{tot}} \Psi_{\text{tot}} = 0$, where $\Psi_{\text{tot}}(a, \phi, \mathcal{Q}_{\mathbf{k}})$ is the quantum state including both background and scalar perturbation degrees of freedom. The Hamiltonian $\hat{H}_{\text{tot}} = \hat{H}_{\text{hom}} + \hat{H}_{\text{pert}}$ contains a background term $\hat{H}_{\text{hom}} = \left(\hat{\Theta} - \partial_{\phi}^{2}\right)/2$ corresponding to the LQC Hamiltonian constraint for the homogeneous FLRW spacetime (as reviewed in the Handbook's previous chapter on homogeneous LQC), with $\hat{\Theta}$ being the gravitational part of $\hat{H}_{\text{hom}}$, while $\hat{H}_{\text{pert}}$ is the Hamiltonian for the scalar perturbations (for the extension to tensor modes, see [9]).

更具体地说，遵循 LQG 的原理，修饰度规方法中微扰的动力学是从惠勒-德维特方程 $\hat{H}_{\text{tot}}\Psi_{\text{tot}} = 0$ 出发推导得到的, 其中 $\Psi_{\text{tot}}(a, \phi, \mathcal{Q}_{\mathbf{k}})$ 是同时包含背景和标量微扰自由度的量子态。哈密顿量 $\hat{H}_{\text{tot}} = \hat{H}_{\text{hom}} + \hat{H}_{\text{pert}}$ 包含背景项 $\hat{H}_{\text{hom}} = \left(\hat{\Theta} - \partial_{\phi}^{2}\right)/2$，对应齐性 FLRW 时空的 LQC 哈密顿约束 (本手册此前齐性 LQC 章节已有综述)，其中 $\hat{\Theta}$ 是 $\hat{H}_{\text{hom}}$ 的引力部分，$\hat{H}_{\text{pert}}$ 是标量微扰的哈密顿量 (关于张量模式的扩展见 [9])。

After deparameterization and interpreting ϕ as time, $\hat{H}_{\text{tot}}\Psi_{\text{tot}} = 0$ can be written as a Schrodinger equation

去参数化并将 ϕ 诠释为时间后，$\hat{H}_{\text{tot}}\Psi_{\text{tot}} = 0$ 可以写为薛定谔方程的形式

$$-i\partial_{\phi}\Psi_{\text{tot}} = \left|\hat{\Theta} - 2\hat{H}_{\text{pert}}\right|^{\frac{1}{2}}\Psi_{\text{tot}}. \tag{5}$$

Perturbative methods can be used to solve this equation, following steps analogous to those used for the classical theory. First, it is assumed the background geometry is not affected by the perturbations, so the total wave function has a product form $\Psi_{\text{tot}}(a, \phi, \mathcal{Q}_{\mathbf{k}}) = \Psi_{\text{hom}}(a, \phi) \otimes \Psi_{\text{pert}}(a, \phi, \mathcal{Q}_{\mathbf{k}})$. Second, the operator $\hat{\Theta}$ is interpreted as the Hamiltonian of the "heavy" degree of freedom and $\hat{H}_{\text{pert}}$ as the Hamiltonian of the light degree of freedom; using this to expand the square root in (5) gives [32]

微扰方法可用于求解该方程，步骤与经典理论所用步骤类似。首先，假设背景几何不受微扰影响，因此总波函数具有乘积形式 $\Psi_{\text{tot}}(a, \phi, \mathcal{Q}_{\mathbf{k}}) = \Psi_{\text{hom}}(a, \phi) \otimes \Psi_{\text{pert}}(a, \phi, \mathcal{Q}_{\mathbf{k}})$。其次，算符 $\hat{\Theta}$ 被解释为"重"自由度的哈密顿量，$\hat{H}_{\text{pert}}$ 被解释为"轻"自由度的哈密顿量；利用这一点展开式 (5) 中的平方根可得 [32]

$$-i\left(\left(\partial_{\phi}\Psi_{\text{hom}}\right) \otimes \Psi_{\text{pert}} + \Psi_{\text{hom}} \otimes \left(\partial_{\phi}\Psi_{\text{pert}}\right)\right) = \sqrt{\hat{\Theta}}\Psi_{\text{hom}} \otimes \Psi_{\text{pert}} - \hat{H}_{\text{pert}}\left(\Psi_{\text{hom}} \otimes \Psi_{\text{pert}}\right).$$

Note that $\hat{\Theta}$ does not act on perturbations, but $\hat{H}_{\text{pert}}$ acts on both background and perturbation states, as it contains background as well as perturbation operators. The third step is to choose Ψ_{hom} so it satisfies the background quantum equation $-i\partial_{\phi}\Psi_{\text{hom}} = \sqrt{\hat{\Theta}}\Psi_{\text{hom}}$. This makes the first term in the left and right sides of the previous equation to cancel out, and the remaining quantum dynamics

注意 $\hat{\Theta}$ 不对微扰作用, 但 $\hat{H}_{\text{pert}}$ 同时对背景态和微扰态作用, 因为它同时包含背景算符与微扰算符。第三步是选择 Ψ_{hom} 使其满足背景量子方程 $-i\partial_{\phi}\Psi_{\text{hom}} = \sqrt{\hat{\Theta}}\Psi_{\text{hom}}$。这使得前一方程左右两侧的第一项相互抵消，剩余的量子动力学

$$\Psi_{\text{hom}} \otimes \left(i\partial_{\phi}\Psi_{\text{pert}}\right) = \hat{H}_{\text{pert}}\left(\Psi_{\text{hom}} \otimes \Psi_{\text{pert}}\right), \tag{6}$$

can be used to solve for $\Psi_{\text{pert.}}$ Since the left side of this last equation is proportional to Ψ_{hom}, it follows that so is the right side. Due to this property, no information is lost when taking the inner product of this equation with Ψ_{hom}, which gives

可用于求解 $\Psi_{\text{pert.}}$。由于最后这个方程的左侧与 Ψ_{hom} 成正比，因此右侧也必然与 Ψ_{hom} 成正比。基于这一性质，对该方程取与 Ψ_{hom} 的内积时不会丢失信息，由此可得

$$i\partial_\phi \Psi_{\text{pert}} = \left\langle \widehat{H}_{\text{pert}} \right\rangle \Psi_{\text{pert}}, \tag{7}$$

where the expectation value is taken in the state Ψ_{hom}.

其中期望是在态 Ψ_{hom} 中计算的。

Interestingly, the previous equation shows that, at leading order, the evolution of perturbations propagating on a quantum FLRW geometry Ψ_{hom} is described by replacing the background operators in the Hamiltonian $\widehat{\mathcal{C}}_2^{(\mathcal{Q})}$ by their expectation value in the state Ψ_{hom}. This, in turn, allows us to write the equations of motion in the Heisenberg picture, producing [11]

有趣的是，上述方程表明，在领头阶，传播在量子 FLRW 几何 Ψ_{hom} 上的微扰演化可以通过将哈密顿量 $\widehat{\mathcal{C}}_2^{(\mathcal{Q})}$ 中的背景算符替换为它在态 Ψ_{hom} 中的期望值来描述。这反过来又允许我们写出海森堡绘景下的运动方程，得到 [11]

$$\widehat{\mathcal{Q}}_{\mathbf{k}}'' + 2\frac{\tilde{a}'}{\tilde{a}}\widehat{\mathcal{Q}}_{\mathbf{k}}' + \left(k^2 + \widetilde{\mathcal{U}}_d(\tilde{\eta})\right)\widehat{\mathcal{Q}}_{\mathbf{k}} = 0, \tag{8}$$

where the effective conformal time $\tilde{\eta}$ is defined by its relation to the internal time ϕ

其中有效共形时间 $\tilde{\eta}$ 由它与内时间 ϕ 的关系定义

$$\mathrm{d}\tilde{\eta} \ := \ \tilde{a}^2(\phi)\left\langle \widehat{\Theta}^{-\frac{1}{2}} \right\rangle \mathrm{d}\phi, \tag{9}$$

and

且

$$\tilde{a} \ := \left(\frac{\langle \widehat{\Theta}^{-\frac{1}{4}}\hat{a}^4(\phi)\widehat{\Theta}^{-\frac{1}{4}}\rangle}{\langle \widehat{\Theta}^{-\frac{1}{2}}\rangle}\right)^{1/4}, \ \widetilde{\mathcal{U}}_d = \frac{\langle \widehat{\Theta}^{-\frac{1}{4}}\hat{a}^2\widehat{\mathcal{U}}\hat{a}^2\widehat{\Theta}^{-\frac{1}{4}}\rangle}{\langle \widehat{\Theta}^{-\frac{1}{4}}\hat{a}^4\widehat{\Theta}^{-\frac{1}{4}}\rangle}. \tag{10}$$

The presence of the operator $\widehat{\Theta}$ in these expressions originates from the lapse function N_ϕ associated with evolution in internal time ϕ; this lapse is implicitly included in the Hamiltonian $\widehat{H}_{\text{pert}}$ in Eq. (7); see [11] for details. Note that whenever there are factor ordering ambiguities, a symmetric order has been chosen. Another ambiguity concerns the form of $\widehat{\mathcal{U}}$. Before quantization, it is possible to use the classical Friedman equations to rewrite $\mathcal{U}$ (e.g., replacing H^2 by $3/\kappa\rho$, with ρ the energy density of the scalar field); but since the Friedman equations are modified in LQC, rewritings of $\mathcal{U}$ of this type produce inequivalent expressions for the quantum operator $\widehat{\mathcal{U}}$. This ambiguity is intrinsic to quantum theories with constraints, and it will arise on several occasions throughout this chapter.

这些表达式中算符 $\widehat{\Theta}$ 的存在源于与内时间 ϕ 演化关联的时移函数 N_ϕ；该时移被隐含包含在式 (7) 的哈密顿量 $\widehat{H}_{\rm pert}$ 中；详见文献 [11]。请注意，凡存在因子排序歧义的地方，我们都选择了对称排序。另一处歧义与 $\widehat{\mathcal{U}}$ 的形式有关。量子化之前，可以利用经典弗里德曼方程重写 $\mathcal{U}$ (例如，将 H^2 替换为 $3/\kappa\rho$，其中 ρ 是标量场的能量密度)；但由于弗里德曼方程在圈量子宇宙学中被修改，这类对 $\mathcal{U}$ 的重写会给出量子算符 $\widehat{\mathcal{U}}$ 的不等价表达式。这种歧义是带约束量子理论的内禀性质，在本章中会多次出现。

A key result is that (8) has the same general form as the semi-classical equation (3) for $\widehat{\mathcal{D}}$. Although the framework is conceptually very different, with a background described by a quantum LQC state, at leading order in perturbations, the evolution (8) is identical to a field theory on a smooth FLRW metric with the effective, quantum-corrected line element $ds^2 = \tilde{a}^2(\tilde{\eta})\left(-d\tilde{\eta}^2 + d\mathbf{x}^2\right)$. Finally, in terms of the Mukhanov-Sasaki variable v, the dynamics are

一个核心结论是: 式 (8) 与 $\widehat{\mathcal{D}}$ 的半经典方程 (3) 具有完全相同的一般形式。尽管二者在概念上框架差异很大——此处背景由量子 LQC 态描述，但在微扰的领头阶下，演化方程 (8) 和有效量子修正线元为 $ds^2 = \tilde{a}^2(\tilde{\eta})\left(-d\tilde{\eta}^2 + d\mathbf{x}^2\right)$ 的光滑 FLRW 度规上的场论完全一致。最后，以穆哈诺夫-佐佐木变量 v 表示的动力学为

$$\hat{v}''_{\mathbf{k}} + \left(k^2 - \frac{\tilde{a}''}{\tilde{a}} + \widetilde{\mathcal{U}}_d\right)\hat{v}_{\mathbf{k}} = 0. \tag{11}$$

These results merit a brief discussion. In LQC, the background homogeneous geometry is quantum and described by a wave function $\Psi_{\rm hom}$. There is no smooth geometry on which other fields propagate and no a priori notion of light-cones. Instead, the boundaries of causal propagation emerge from the full quantum dynamics: the propagation of $\mathcal{Q}$ is exactly that of fields on a globally hyperbolic FLRW spacetime with metric tensor $\tilde{g}_{ab}$, and the equations of motion are generally covariant and locally Lorentz invariant. In other words, a quantum field theory on a FLRW geometry emerges from the quantum geometry $\Psi_{\rm tot}$.

这些结果值得简要讨论。在 LQC 中，背景均匀几何是量子化的，由波函数 $\Psi_{\rm hom}$ 描述。不存在可供其他场传播的光滑几何，也没有先验的光锥概念。相反，因果传播的边界由完整量子动力学导出: $\mathcal{Q}$ 的传播与整体双曲 FLRW 时空上度规张量为 $\tilde{g}_{ab}$ 的场传播完全一致，且运动方程满足广义协变性与局域洛伦兹不变性。换句话说，FLRW 几何上的量子场论从量子几何 $\Psi_{\rm tot}$ 中自然涌现。

The metric $\tilde{g}_{ab}$ is commonly called the effective dressed metric [32]; beyond approximating the physical background geometry, it encodes all the information contained in $\Psi_{\rm hom}$ that the test field $\mathcal{Q}$ is sensitive to. This is what the adjective ”effective” refers to. On the other hand, $\tilde{g}_{ab}$ is also called ”dressed” because it depends not only on the mean value of $\Psi_{\rm hom}$ but also on some of its quantum fluctuations.

度规 $\tilde{g}_{ab}$ 通常被称为有效修饰度规 [32]；它除了近似物理背景几何之外，还编码了 $\Psi_{\rm hom}$ 中所有能被测试场 $\mathcal{Q}$ 感知的信息。这就是“有效”一词的含义。另一方面，$\tilde{g}_{ab}$ 被称为“修饰”，是因为它不仅依赖 $\Psi_{\rm hom}$ 的平均值，还依赖其部分量子涨落。

At a practical level, to evolve perturbations on a quantum FLRW geometry $\Psi_{\rm hom}$, it is sufficient to compute the components of $\tilde{g}_{ab}$ from $\Psi_{\rm hom}$ and then proceed exactly as in standard quantum field theory in curved spacetimes.

在实用层面，要演化量子 FLRW 几何 Ψ_{hom} 上的微扰，只需从 Ψ_{hom} 计算出 $\tilde{g}_{ab}$ 的分量，之后的步骤就和弯曲时空标准量子场论完全一致。

If the state Ψ_{hom} is sharply peaked on a classical trajectory at late times (i.e., if $\Delta a/\langle a\rangle \ll 1$ at late time, where Δa is the quantum dispersion of $\hat{a}$), then the scale factor $\tilde{a}$ of the dressed metric reduces to a solution of the effective equations of LQC, discussed in the Handbook's previous chapter on homogeneous LQC. In this case, the dressed metric approach becomes very simple and is formally identical to the semi-classical theory of cosmological perturbations, except with the scale factor replaced by a solution of the effective equations of LQC. Since the effective equations become indistinguishable from the classical Friedmann equations far from the Planck regime, this implies that the quantum field theory in quantum spacetimes just described reduces to standard quantum field theory on classical spacetimes in that regime.

如果态 Ψ_{hom} 在晚期尖锐峰化在经典轨迹上 (即当晚期满足 $\Delta a/\langle a\rangle \ll 1$，其中 Δa 是 $\hat{a}$ 的量子弥散)，那么修饰度规的标度因子 $\tilde{a}$ 会退化为 LQC 有效方程的解，该内容已在本手册之前关于均匀 LQC 的章节中讨论过。这种情况下，修饰度规方法变得非常简单，形式上和宇宙学微扰的半经典理论完全一致，区别仅在于标度因子被替换为 LQC 有效方程的解。由于远离普朗克能区时，有效方程和经典弗里德曼方程无法区分，这意味着前文所述量子时空上的量子场论，在该能区会退化为经典时空上的标准量子场论。

The computation of the dressed metric $\tilde{g}_{ab}$ can be formally extended to background quantum states Ψ_{hom} that are not necessarily sharply peaked, but if quantum fluctuations are large, then there arise infrared divergences due to "long tails" in Ψ_{hom} [133, 135] . This is reminiscent of the infrared divergences that arise in the calculation of the S-matrix in QED. One can envisage mechanisms for mathematically taming these divergences, but a better physical understanding of them is needed first.

修饰度规 $\tilde{g}_{ab}$ 的计算可以形式上推广到不一定尖锐峰化的背景量子态 Ψ_{hom}，但如果量子涨落很大，Ψ_{hom} [133, 135] 中的“长拖尾”会引发红外发散。这让人联想到量子电动力学中计算 S 矩阵时出现的红外发散。目前已经可以设想出从数学上控制这些发散的机制，但首先需要对它们有更完善的物理理解。

The perturbative approach reviewed here, as well as the hybrid quantization described next, has been criticized for neglecting possible quantum corrections to the definition of gauge-invariant perturbations while including quantum corrections for the background degrees of freedom, which may entail a violation of general covariance [50] (although note general covariance is broken in all perturbative approaches, particularly once backreaction is neglected). As explained in assumption (i) at the beginning of the subsection "Dressed Metric Approach", working with the classical gauge-invariant variables consists in assuming that quantum corrections to gauge transformations are subleading compared to the impact of modified background dynamics on the perturbations. It would be desirable though to test this assumption from the viewpoint of full LQG.

本文回顾的这种微扰方法，以及下文将要介绍的混合量子化方法，一直被批评为: 在对背景自由度计入量子修正的同时，却忽略了规范不变扰动定义中可能存在的量子修正，这可能会导致违背广义协变性 [50](不过需要注意，所有微扰方法都会破坏广义协变性，尤其是在忽略反作用的情况下更是如此)。正如“修正度规方法”小节开头假设 (i) 所解释的，使用经典规范不变变量即代表我们假设: 相比修正后背景动力学对扰动的影响，规范变换的量子修正属于次 Leading 阶。不过，从完整圈量子引力 (LQG) 的角度对这一假设进行检验，仍是有待完成的工作。

Finally, recall that an important assumption underlying this formalism is the absence of significant backreaction of the perturbations on the homogeneous geometry. This assumption can be checked by comparing the expectation value of the renormalized energy and pressure of perturbations with the background contribution [10, 11] and also by going to the next-to-leading order in perturbations [12]. The first approach suffers from the standard ambiguities in defining the renormalized energy-momentum tensor in quantum field theory in curved space-times [197]; using adiabatic regularization it has been shown that backreaction is negligible throughout the evolution [11]. Similarly, following the second approach, the contribution of perturbations at next-to-leading order is negligible; see the subsection ”Non-Gaussianity” for details.

最后需要重申，该形式体系的一个重要基础假设是: 扰动对均匀几何不存在显著的反作用。这一假设可以通过两种方式检验: 一是将扰动重整化能量与压强的期望值和背景贡献 [10, 11] 对比，二是计算到扰动的次 Leading 阶 [12]。第一种方法存在弯曲时空量子场论中定义重整化能量动量张量的标准模糊性问题 [197]；但采用绝热正则化方法的研究表明，反作用在整个演化过程中都可以忽略 [11]。同样，采用第二种方法的研究也得到次 Leading 阶扰动贡献可忽略的结论；详见“非高斯性”小节。

Hybrid Quantization

混合量子化

Similar to the dressed metric approach, the hybrid quantization also adopts the philosophy of combining a loop quantization of the homogeneous and isotropic universe with a Fock quantization of the inhomogeneous perturbations [105, 106, 164]. However, the strategy in the two approaches is different. In the dressed metric approach, the total Hamiltonian is first split in two parts, namely, the constraint for the background and the quadratic Hamiltonian for perturbations, which are treated separately, analogous to what is done in standard cosmology. On the other hand, in the hybrid approach, the total Hamiltonian is truncated at quadratic order in perturbations, and the background and quadratic Hamiltonians of the truncated system are treated together as a constrained symplectic system, following [124]. In principle, the latter provides a path to include some backreaction of perturbations [99], although a consistent treatment requires going to second order in perturbation theory (since quadratic backreaction from linear perturbations is of the same order as linear backreaction from second-order perturbations) and this has not been worked out explicitly yet. When backreaction is neglected, the two approaches are classically equivalent for linear cosmological perturbations. However, the concrete implementations of the two approaches put forward in the literature so far differ partially because of different choices in factor-ordering ambiguities and in the choice of the definition of the operator $\widehat{\mathcal{U}}$ [84, 119] .

与修正式度规方法类似，混合量子化同样遵循将齐次各向同性宇宙的圈量子化与非齐次微扰的福克量子化结合的思路 [105, 106, 164]。但两种方法的处理策略不同。在修正式度规方法中，总哈密顿量首先被拆分为两部分，即背景约束和微扰的二次哈密顿量，二者分开处理，这和标准宇宙学中的做法类似。而在混合方法中，总哈密顿量在微扰的二阶处截断，遵循文献 [124] 的工作，将截断后系统的背景哈密顿量与二次哈密顿量作为一个约束辛系统整体处理。原则上，后者为包含微扰的部分反作用提供了途径 [99]，不过自洽的处理需要推进到微扰理论的二阶 (因为线性微扰的二次反作用与二阶微扰的线性反作用同阶)，这一点目前还没有得到明确的研究。当忽略反作用时，两种方法在线性宇宙学微扰下经典等价。但迄今为止文献中两种方法的具体实现存在部分差异，这源于因子排序不确定性的不同选择，以及算符 $\hat{\mathcal{U}}$ [84, 119] 定义的不同选择。

In particular, when neglecting the backreaction, in the hybrid approach, the linearized equation for scalar perturbations is [98]

特别地，当忽略反作用时，混合方法中标量微扰的线性化方程为 [98]

$$v_k'' + \left(k^2 - \frac{4\pi G}{3} a^2 (\rho - 3P) + \mathcal{U}_h\right) v_k = 0, \tag{12}$$

where

其中

$$\mathcal{U}_h = a^2 \left[V_{\phi\phi}(\phi) + 48\pi G V(\phi) - \frac{48\pi G}{\rho} V^2(\phi) + \frac{6a'\phi'}{a^3\rho} V_{\phi}(\phi)\right]. \tag{13}$$

The functions ρ and P are the energy density and pressure of the homogeneous universe; for a scalar field with a potential $V(\phi)$, we have $\rho = (\phi'/a)^2/2 + V(\phi)$ and $P = \rho - 2V(\phi)$. The potential $\mathcal{U}_h$ and its analog in the dressed metric, $\mathcal{U}_d$, have the same classical limit; but they are different in LQC close to the bounce (and the same is true for $(4\pi G/3)\, a^2 (\rho - 3P)$ and a''/a). This is an example of the ambiguity in the choice for the potential $\mathcal{U}$ for the perturbations: different combinations are possible that give the correct classical limit.

函数 ρ 和 P 分别是齐次宇宙的能量密度和压强；对于带势 $V(\phi)$ 的标量场，我们有 $\rho = (\phi'/a)^2/2 + V(\phi)$ 和 $P = \rho - 2V(\phi)$ 。势 $\mathcal{U}_h$ 和它在修正式度规中的对应项 $\mathcal{U}_d$ 具有相同的经典极限，但在反弹附近的圈量子宇宙学中二者并不相同 ($(4\pi G/3)\, a^2 (\rho - 3P)$ 和 a''/a 也是如此)。这就是微扰势 $\mathcal{U}$ 选择存在不确定性的一个例子: 可以得到不同的组合，它们都能给出正确的经典极限。

It is interesting that the power spectra for the scalar and tensor perturbations calculated in these two approaches are quite similar [11, 16, 67, 85, 99, 208, 211], despite the fact that the effective potentials near the bounce can be different [98, 129].

有意思的是，尽管反弹附近的有效势可能不同 [98, 129]，但两种方法计算得到的标量微扰和张量微扰功率谱却十分相似 [11, 16, 67, 85, 99, 208, 211]。

The similarity of the results for physical observables, in spite of these differences, is mainly because the current observational frequency band observed in the CMB today corresponds to ultraviolet frequencies at the time of the quantum bounce, for which the effects of the potential are negligibly small; this is true both for the hybrid [150, 208] and the dressed metric approaches [150, 211]. For further discussions on this point, see [93, 145] and the subsection ”Inflation in the Dressed Metric and Hybrid Approaches”.

尽管存在上述差异，但物理可观测量的结果之所以相似，主要是因为如今在宇宙微波背景中观测到的当前观测频段对应量子反弹时的紫外频率，对于这类频率，势的效应可以忽略不计；这一点对混合方法 [150, 208] 和修正式度规方法 [150, 211] 都成立。关于这一点的更多讨论，参见文献 [93, 145] 以及“修正式度规方法与混合方法中的暴胀”小节。

Separate Universe Loop Quantization

分离宇宙圈量子化

Another approach to cosmological perturbation theory in LQC is based on an adaptation of the ”separate universe” framework [178, 199]. The basic idea underlying this approach is that ”derivative” terms are negligible for long-wavelength perturbations. Specifically, for a Fourier mode whose wavelength is greater than the Hubble radius, $k^2 \ll z''/z$ in the equation of motion (4) and therefore the k^2 term can safely be neglected.

LQC 中宇宙微扰理论的另一种方法基于对“分离宇宙”框架的改编 [178, 199]。该方法的核心思路是: 长波长微扰的“导数项”可忽略。具体来说，对于波长大于哈勃半径的傅里叶模式，运动方程 (4) 中的 $k^2 \ll z''/z$ ，因此 k^2 项可以安全忽略。

Due to this simplification, it is possible to calculate LQC corrections for long-wavelength modes in a relatively direct manner [202,205]. First, introduce a spatial discretization, such that the lattice spacing is greater than $\sqrt{|z/z''|}$; clearly this only captures long-wavelength perturbations. Note that neglecting the k^2 term in Fourier space implies neglecting derivatives in position space, and on a lattice this implies neglecting interaction terms between neighboring cells in the lattice. Second, for each cell in the discretization, the scale factor $a(\mathbf{x})$ is uniform, and therefore the spacetime geometry of each cell corresponds to a homogeneous spacetime. Working in the longitudinal gauge, the spatial metric has the form $q_{ab} = \mathrm{diag}\left(a(\mathbf{x})^2, a(\mathbf{x})^2, a(\mathbf{x})^2\right)$ with $a(\mathbf{x}) = a(1 - \psi(\mathbf{x}))$, which (for each cell) is exactly the flat FLRW metric. Of course, the scale factor $a(\mathbf{x})$ and the energy density in each cell in the lattice cannot vary too much from one cell to another if the discretization is to describe small perturbations on a homogeneous background. Finally, the dynamics in each cell are generated by precisely the Hamiltonian constraint of the flat FLRW spacetime (with no new terms since interactions are being neglected for the long-wavelength modes being studied here) so the standard loop quantization for FLRW (reviewed in the Handbook’s previous chapter on homogeneous LQC) can be applied, without any modifications, to each cell. This process gives a loop quantization for long-wavelength scalar perturbations.

通过这一简化，可以相对直接地计算长波长模式的 LQC 修正 [202,205]。首先，引入空间离散化，格点间距大于 $\sqrt{|z/z''|}$；显然这仅能捕获长波长微扰。注意，在傅里叶空间忽略 k^2 项等价于在位置空间忽略导数，在格点上等价于忽略格点相邻单元之间的相互作用项。其次，离散化后的每个单元中，标度因子 $a(\mathbf{x})$ 是均匀的，因此每个单元的时空几何对应一个均匀时空。取纵 gauge，度规形式为 $q_{ab} = \operatorname{diag}\left(a(\mathbf{x})^2, a(\mathbf{x})^2, a(\mathbf{x})^2\right)$，其中 $a(\mathbf{x}) = a(1-\psi(\mathbf{x}))$，(对每个单元而言) 这正是平坦 FLRW 度规。当然，如果离散化要描述均匀背景上的小微扰，格点中每个单元的标度因子 $a(\mathbf{x})$ 和能量密度在单元间的变化不能过大。最后，每个单元的动力学恰好由平坦 FLRW 时空的哈密顿约束生成 (由于本文研究的长波长模式已经忽略相互作用，因此没有额外项)，因此 FLRW 的标准圈量子化 (本手册此前关于均匀 LQC 的章节已综述) 无需修改即可应用于每个单元。这一过程就得到了长波长标量微扰的圈量子化。

When quantum fluctuations are small, the dynamics of the long-wavelength scalar perturbations are well approximated by the semi-classical equation

当量子涨落很小时，长波长标量微扰的动力学可以被半经典方程很好地近似为

$$v_k'' - \frac{z''}{z} v_k = 0 \tag{14}$$

where v is the Mukhanov-Sasaki variable introduced in (4). Note that this equation clearly has the correct classical limit.

其中 v 是 (4) 中引入的穆哈诺夫-佐佐木变量。可见该方程显然具有正确的经典极限。

Further, this result fixes some ambiguities. In classical general relativity, it is possible to use the Friedman equation to replace, e.g., H , by $\sqrt{8\pi G\rho/3}$ in the denominator of z ; however, the relation between H and ρ changes in LQC. Therefore, it is necessary to determine what form of z is the correct one for LQC-this calculation based on the separate universe approach gives the preferred form of $z = a\dot{\phi}/H$ as the correct choice for the potential for scalar perturbations in LQC. (Note that both z and v diverge at the bounce where $H = 0$. This is not a problem with the dynamics but rather signals that v is not a good variable for perturbations at the bounce-instead, it is necessary to use another variable, say the comoving curvature perturbation $\mathcal{R}$, to describe scalar perturbations at the bounce point.)

此外，这一结果还消除了部分歧义。在经典广义相对论中，可以利用弗里德曼方程，例如将 z 分母中的 H 替换为 $\sqrt{8\pi G\rho/3}$；但在 LQC 中，H 和 ρ 的关系会发生变化。因此需要确定 z 的哪种形式在 LQC 中是正确的——这种基于分离宇宙方法的计算得出，优先选择 $z = a\dot{\phi}/H$ 的形式作为 LQC 中标量微扰势的正确形式。(注意 z 和 v 在反弹处都会发散，此时 $H = 0$ 。这不是动力学的问题，反而说明 v 不是反弹处微扰的好变量——需要改用另一个变量，例如共动曲率微扰 $\mathcal{R}$，来描述反弹点的标量微扰。)

As explained above, the separate universe approach only focuses on super-Hubble modes and neglects shorter-wavelength modes, so a natural question is whether it may be possible to extend this approach to shorter wavelengths. In particular, at the bounce the curvature radius is $\sim \ell_{\mathrm{Pl}}$, so is it possible to describe perturbations with a wavelength shorter than ℓ_{Pl} by generalizing this approach? (This question is closely related to the trans-Planckian problem that will be discussed in more detail in the subsection "Trans-Planckian Modes".)

如上所述，分离宇宙方法仅聚焦于超哈勃模式，忽略了短波长模式，因此一个自然的问题是: 能否将该方法推广到更短的波长？特别地，反弹处的曲率半径为 $\sim \ell_{\mathrm{Pl}}$，那么是否可以通过推广该方法来描述波长小于 ℓ_{Pl} 的微扰？(该问题与“跨普朗克模式”小节将要详细讨论的跨普朗克问题密切相关。)

It turns out that this is not possible, for the following reason. In such a quantum theory, for a given state to correspond to a cosmological spacetime with small perturbations, it is necessary that the expectation value of any operator $\widehat{\mathcal{O}}$ evaluated in any cell be close to the expectation value of $\widehat{\mathcal{O}}$ averaged over all cells. It can be shown that this condition implies that the volume of each cell in the lattice must be much larger than ℓ_{Pl}^3 [202], so the shortest Fourier mode that can be resolved in this approach for states with small perturbations has a wavelength greater than ℓ_{Pl}. In short, the approach of discretizing a cosmological spacetime with small perturbations on a lattice works well, but only for perturbations whose wavelengths are always much greater than ℓ_{Pl}: it is impossible to resolve trans-Planckian modes.

事实证明，由于下述原因，这是无法实现的。在这类量子理论中，若要让给定量子态对应带微扰的宇宙学时空，就必须满足: 任何算符 $\widehat{\mathcal{O}}$ 在任意格胞内计算得到的期望值，都接近 $\widehat{\mathcal{O}}$ 对所有格胞求平均后的期望值。可以证明，这一条件要求格点中每个格胞的体积必须远大于 ℓ_{Pl}^3 [202]，因此对于带小微扰的态，该方法能分辨的最短傅里叶模式的波长也大于 ℓ_{Pl}。简而言之，带小微扰的宇宙学时空的格点离散化方法效果很好，但它仅适用于波长始终远大于 ℓ_{Pl} 的微扰: 无法分辨跨普朗克模式。

Note that this result does not imply that there must be a Planckian cutoff: perhaps a discretization on a lattice is not an appropriate approximation for trans-Planckian modes, or perhaps it is necessary to allow fluctuations to be large at trans-Planckian scales (although in this case it would be necessary to go beyond linear perturbation theory for these modes). In any case, it does not seem possible to directly generalize the separate universe approach to trans-Planckian modes.

请注意，这一结果并不意味着必然存在普朗克截断: 可能格点离散化本身就不是跨普朗克模式的合适近似，也可能跨普朗克尺度下涨落本身就很大 (不过这种情况下就必须对这些模式超出线性微扰理论的框架处理)。无论如何，分宇宙方法似乎都无法直接推广到跨普朗克模式。

On the other hand, although this has not yet been done, it does seem likely that the separate universe framework could be extended to tensor modes. This could be done by considering a separate universe framework applied to a lattice of Bianchi I spacetimes (for the loop quantization of the Bianchi I spacetime, see [36]), with the metric $g_{ab} = \mathrm{diag}\left(-1, a_1(t)^2, a_2(t)^2, a_3(t)^2\right)$ in each cell of the lattice. Taking $a_3 = a$ and setting $a_1 = a(1+h)$ and $a_2 = a(1-h)$ with $h \ll 1$, then h captures a gravitational wave in the $+$ polarization moving in the x_3 direction on a flat FLRW background. Using the same steps as for scalar perturbations, it should be possible to derive the equations of motion for this particular tensor mode perturbation. But this result can immediately be generalized, since the same equations should hold for both polarizations of the tensor modes and for all directions.

另一方面，尽管相关工作尚未完成，但分宇宙框架很可能可以拓展到张量模式。具体可以通过将分宇宙框架应用于 Bianchi I 时空格点来实现 (Bianchi I 时空的圈量子化参见文献 [36])，格点每个胞内的度规为 $g_{ab} = \mathrm{diag}\left(-1, a_1(t)^2, a_2(t)^2, a_3(t)^2\right)$。取 $a_3 = a$，令 $a_1 = a(1+h)$ 和 $a_2 = a(1-h)$ 满足 $h \ll 1$，则 h 就描述了平坦 FLRW 背景下沿 x_3 方向传播的 + 极化引力波。遵循标量微扰的相同步骤，应当可以推导出该特定张量模式微扰的运动方程。而且这一结果可以直接推广，因为相同的方程对张量模式的两种极化、所有传播方向都成立。

In summary, the separate universe approach to cosmological perturbations can be successfully adapted to LQC, giving a loop quantization of long-wavelength scalar modes. This framework has an important strength, as well as a major limitation. Its strength is that this is a loop quantization for these cosmological perturbation modes, not a Fock quantization as is done in the hybrid and dressed metric approaches. On the other hand, the limitation is significant, since it can only be applied to long-wavelength (super-Hubble) modes. As a result, it cannot be used to study inflation, since for inflationary models the observationally relevant modes start with a wavelength much smaller than the Hubble radius. Nonetheless, as shall be discussed later, this approach can be used to study some alternatives to inflation like ekpyrosis and the matter bounce.

综上所述，宇宙学微扰的分宇宙方法可以成功适配圈量子宇宙学，实现长波长标量模式的圈量子化。该框架有一个重要优势，也存在一个显著局限。它的优势在于，这是针对这些宇宙学微扰模式的圈量子化，而非混合方法和装扮度规方法中采用的福克量子化。而另一方面，它的局限十分明显: 该方法仅能应用于长波长 (超哈勃) 模式。因此，它无法用于研究暴胀，因为在暴胀模型中，观测相关的模式初始波长远小于哈勃半径。不过我们在后文会讨论，该方法可以用于研究一些暴胀的替代模型，比如火劫模型和物质反弹模型。

Anomaly-Free Effective Dynamics

无反常有效动力学

In the literature, the anomaly-free effective dynamics approach is also referred to as the deformed (or closed) algebra approach. It is a semi-classical approach, in which both homogeneous and inhomogeneous parts of the universe are described by an effective metric with the Lorentz signature, very much like what have been done in classical modern cosmology, with the only exception that quantum corrections from both gravitational and matter sectors (to their leading order) are taken into account. These corrections are obtained by adding quantum counterterms into the classical Hamiltonian constraint, so that the deformed constraint algebra is still closed and anomaly-free. The latter is essential and guarantees that such obtained effective theory is still generally covariant [190].

在文献中，无反常有效动力学方法也被称为形变 (或闭合) 代数方法。它是一种半经典方法，其中宇宙的均匀部分和非均匀部分都由具有洛伦兹符号的有效度规描述，这与经典现代宇宙学的研究方式非常相似，唯一的区别是它计入了引力领域和物质领域的量子修正 (取领头阶)。这些修正通过在经典哈密顿约束中添加量子抵消项得到，从而使形变后的约束代数仍然保持闭合且无反常。后一点至关重要，它保证了这样得到的有效理论仍然满足广义协变性 [190]。

The anomaly-free effective approach is motivated by results in homogeneous LQC showing that for sharply peaked states, there exists an effective line element that provides an excellent approximation to the full quantum state [33, 97, 189]. Although it is not known if a similar effective description exists for perturbations, general arguments suggest that such a description should be possible for long-wavelength perturbations (but not perturbations with a wavelength $\lesssim \ell_{\mathrm{Pl}}$) [51,177].

无反常有效方法的动机来自均匀圈量子宇宙学的研究结果: 对于尖峰态，存在一个有效线元可以对完整量子态给出极好的近似 [33, 97, 189]。虽然目前尚不清楚微扰是否存在类似的有效描述，但一般性论证表明，长波长微扰应当可以采用这种描述 (但波长为 $\lesssim \ell_{\mathrm{Pl}}$ 的微扰除外)[51,177]。

In this approach no wave functions are involved, and the usual techniques of classical cosmology can be applied here. In particular, it is possible to work in a general gauge (while the dressed metric and hybrid approaches pick a classically gauge-invariant variable before quantization, and the separate universe approach works in the longitudinal gauge). On the other hand, it is not clear to what extent the inclusion of quantum counterterms in the constraints is unique or not. Finally, although it is in principle possible to include quantum fluctuations in the background spacetime perturbatively [49], in practice quantum fluctuations are often ignored in this approach.

该方法不涉及波函数，可以直接应用经典宇宙学的常用技术。具体来说，它可以在任意规范下工作 (而穿衣度规方法和混合方法会在量子化之前选取一类经典规范不变量，分离宇宙方法则在纵向规范下工作)。另一方面，目前尚不清楚在约束中加入量子抵消项的方式在多大程度上是唯一的。最后，尽管原则上可以微扰地计入背景时空的量子涨落 [49]，但实际上该方法通常会忽略量子涨落。

To describe the deformed algebra approach in more detail, consider the classical constraint algebra

为了更详细地描述形变代数方法，我们考虑经典约束代数

$$\{D[M^a], D[N^a]\} = D\left[M^b \partial_b N^a - N^b \partial_b M^a\right], \tag{15}$$

$$\{D[M^a], S[N]\} = S\left[M^b \partial_b N - N \partial_b M^b\right], \tag{16}$$

$$\{S[M], S[N]\} = D\left[q^{ab}(M \partial_b N - N \partial_b M)\right], \tag{17}$$

where N and M are lapse functions, N^a and M^a are shift vectors, q_{ab} denotes the spatial three-dimensional metric, and D and S are the smeared diffeomorphism and Hamiltonian constraints. The constraint algebra is closed and thereby ensures covariance after the (3+1)-dimensional decomposition [190].

其中 N 和 M 是移时函数，N^a 和 M^a 是位移矢量，q_{ab} 代表空间三维度规，D 和 S 是抹平后的微分同胚约束和哈密顿约束。该约束代数是闭合的，因此保证了 (3+1) 维分解之后的协变性 [190]。

When quantum gravitational effects are taken into account, it is expected that this constraint algebra may be modified. Without the full underlying quantum theory, it may be difficult to guess what these modifications may be [88], but to remain covariant the modified algebra should be free of anomalies (i.e., the constraint algebra must remain closed) [190].

当计入量子引力效应后，我们预期该约束代数可能会发生修改。在没有完整基础量子理论的情况下，很难推测这些修改会是什么形式 [88]，但为了保持协变性，修改后的代数应当没有反常 (即约束代数必须保持闭合)[190]。

With this observation in mind, it was found that for linear perturbations on the flat FLRW background, the freedom in the choice of possible deformations can considerably restricted. This was first studied for inverse triad corrections, where under the conditions that (i) the modified effective Hamiltonian constraint must commute with the unchanged (classical) diffeomorphism constraint; (ii) the quantum-corrected constraints must form an anomaly-free Poisson algebra, and (iii) the classical constraints should be recovered in the classical limit, it was shown that it is possible to find anomaly-free effective constraints for scalar [59, 60] , vector [57], and tensor perturbations [58] in closed forms, and the effective equations of motion are derived from these. The observational consequences of these models have been studied, with the result that they can be made consistent with CMB data by properly choosing the parameters of the models [54-56, 212, 213, 216].

基于这一观察，研究发现，对于平坦 FLRW 背景上的线性微扰，可能形变选择的自由度可以被大幅限制。该研究最早针对逆标架修正展开，在以下三个条件下:(i) 修改后的有效哈密顿约束必须与未改变的 (经典) 微分同胚约束对易；(ii) 量子修正后的约束必须构成无反常泊松代数；(iii) 经典极限下可以退化为经典约束，研究表明可以得到标量 [59, 60] 、矢量 [57] 和张量微扰 [58] 的闭合形式无反常有效约束，并由此推导出有效运动方程。这些模型的观测效应已经得到研究，结果表明，只要合理选取模型参数，它们就可以与宇宙微波背景数据一致 [54-56, 212, 213, 216]。

Holonomy corrections were considered next, for scalar, vector, and tensor modes [57, 58, 80, 82, 121, 152, 165, 201] , with the result that the constraint algebra is modified: while the Poisson brackets (15)-(16) are unchanged, the relation (17) becomes

接下来人们研究了整体修正在内的标量、矢量和张量模式 [57, 58, 80, 82, 121, 152, 165, 201] ，结果发现约束代数发生了修改: 泊松括号 (15)-(16) 保持不变，但关系式 (17) 变为

$$\{S[M], S[N]\} = \Omega D\left[q^{ab}\left(M\partial_b N - N\partial_b M\right)\right], \tag{18}$$

where $\Omega \equiv 1 - 2\rho/\rho_c$ (of course, here the smeared Hamiltonian constraint $S[N]$ is the effective version containing holonomy corrections). The observation that $\Omega < 0$ for $\rho_c/2 < \rho \leq \rho_c$ leads to the suggestion of a possible change of signature in the deep ultraviolet regime [61-63] and a possible connection to the no-boundary proposal [52,53], although see also a discussion on the claims of signature change in [206].

其中 $\Omega \equiv 1 - 2\rho/\rho_c$ (当然，此处抹平后的哈密顿约束 $S[N]$ 是包含整体修正的有效版本)。对于 $\rho_c/2 < \rho \leq \rho_c$ ，$\Omega < 0$ 的观测结果引出了一个猜想: 深紫外区可能会发生符号改变 [61-63]，并且可能与无边界 proposal 存在联系 [52,53]，相关讨论也可参见文献 [206] 中关于符号改变命题的讨论。

Once the quantum-corrected effective constraints are known, then the equations of motion are derived following the same steps as in general relativity. The anomaly-free approach gives a construction for effective constraints that include three classes of terms: zeroth order, first order, and second order in the perturbations. The zeroth terms determine the background dynamics with effective Friedman and Raychaudhuri equations, while the first-order terms define the gauge-invariant variables, and the second-order terms generate the dynamics for the perturbations. The equations of motion for the holonomy-corrected scalar perturbations are

[82]

一旦得到量子修正的有效约束，便可遵循广义相对论中的相同步骤推导运动方程。无异常方法为有效约束提供了一种构造，该构造包含三类按扰动划分的项: 零阶项、一阶项和二阶项。零阶项决定背景动力学，给出有效弗里德曼方程与瑞乔杜里方程，一阶项定义规范不变变量，二阶项则生成扰动的动力学。全同态修正标量扰动的运动方程为 [82]

$$v_k''(\eta) + \left(\Omega k^2 - \frac{z''}{z}\right) v_k(\eta) = 0. \tag{19}$$

For the anomaly-free effective dynamics for tensor modes, see [80], and for the inclusion of both holonomy and inverse triad corrections, see [81].

关于张量模的无异常有效动力学，参见文献 [80]；关于同时包含全同态修正与逆三重校正的研究，参见文献 [81]。

The main drawback of these equations is that they ignore quantum fluctuations so they cannot be trusted to evolve trans-Planckian modes (which necessarily have large quantum fluctuations) [162, 206]. (If one ignores this and imposes initial conditions in the remote contracting phase, for the modes that are trans-Planckian during the bounce, one will obtain power spectra that are inconsistent with current observations due to significant amplification of the trans-Planckian modes across the bounce when $\Omega < 0$ [64, 122] .)

这些方程的主要缺陷是忽略了量子涨落，因此无法用于演化跨普朗克模 (这类模必然存在大幅量子涨落)[162, 206]。(如果忽视这一点，在遥远的收缩阶段施加初始条件，对于弹跳过程中为跨普朗克尺度的模，当 $\Omega < 0$ [64, 122] 时，跨普朗克模会在整个弹跳过程中被显著放大，最终得到的功率谱与现有观测结果不一致。)

Because (19) changes from an elliptic to a hyperbolic equation at $\rho = \rho_c/2$, it has been proposed to fix initial conditions at $\rho = \rho_c/2$ [166]; this is called the "silent point." At this point there exists a unique set of initial conditions that give power spectra for the scalar and tensor perturbations that are consistent with the current CMB observations [151].

由于式 (19) 在 $\rho = \rho_c/2$ 处从椭圆方程变为双曲方程，因此有研究提出在 $\rho = \rho_c/2$ 处固定初始条件 [166]，该点被称为“沉默点”。在该点存在唯一一组初始条件，其得到的标量扰动与张量扰动功率谱符合当前 CMB 观测结果 [151]。

Finally, note that in the long-wavelength limit, the effective scalar perturbation equation given by (19) is identical to the result derived using the separate universe approach [202], showing the robustness of this choice for the potential $\mathcal{U}$ in LQC.

最后需要指出，在长波长极限下，式 (19) 给出的有效标量扰动方程与利用分离宇宙方法得到的结果完全一致 [202]，这表明 LQC 中对势 $\mathcal{U}$ 的该选择是稳健的。

Predictions for the CMB

宇宙微波背景的预言

The next step in the program is to use the formalisms described above to make connection with the temperature anisotropies observed in the CMB, as well as to make further predictions testable with current or future observations. Unfortunately, there is no obvious way in which the bounce of LQC alone can generate the scale-invariant temperature anisotropies of the CMB. Consequently, the strategy so far has been to combine LQC with some other mechanism to generate the primordial perturbations, such as inflation or its alternatives, like ekpyrosis or the matter bounce. In this context, the goal of LQC is not to replace these well-known mechanisms but rather to complement and extend them to include Planck scale physics. LQC can possibly add some new features to the primordial perturbations which could be used to test this theoretical framework. This section summarizes recent results in this direction.

该研究的下一步是利用上文描述的形式体系，将其与宇宙微波背景中观测到的温度涨落联系起来，并给出可被当前或未来观测验证的更多预言。遗憾的是，仅靠圈量子宇宙学的反弹，无法直接产生宇宙微波背景的尺度不变温度涨落。因此目前的研究策略是将圈量子宇宙学与其他能够产生原初扰动的机制相结合，比如暴胀，或是其替代模型如火劫模型或物质反弹。在此框架下，圈量子宇宙学的目标并非取代这些成熟机制，而是对它们进行补充和扩展，将普朗克尺度物理纳入其中。圈量子宇宙学有可能为原初扰动引入新的特征，可用于检验这个理论框架。本节总结该方向的最新研究成果。

Inflationary Models in LQC

圈量子宇宙学中的暴胀模型

Among the assumptions on which the inflationary scenario rests, the choice of the initial state for perturbations at the beginning of inflation is particularly important. The inflationary predictions arise by choosing the so-called Bunch-Davies vacuum at the onset of inflation for the range of wavelengths $\lambda = 2\pi/k$ observed in the CMB. These wavelengths are much shorter than the (spacetime) curvature radius at the onset of inflation ($r_{\text{curv}} \approx 1/H$) , and the Bunch-Davies vacuum corresponds to Minkowski-like vacuum fluctuation for these short-wavelength modes, plus sub-leading corrections. Although this premise may sound natural at first, it assumes that these modes have never been excited in the past, before inflation starts. This is a strong assumption given our ignorance about the way inflation starts and what came before. Perturbations would not necessarily reach inflation in the Bunch-Davies vacuum if, for instance, the inflationary phase was preceded by a cosmic bounce: then, observable modes could have exited and re-entered the ”horizon”- the curvature radius, to be more precise-and have been excited during the process. Given a scenario for the pre-inflationary universe, it would be more satisfactory to start the evolution far in the asymptotic past and compute, rather than postulate, the state of perturbations at the onset of slow-roll (although this strategy still requires postulating the initial state for perturbations far in the past). Deviations from Bunch-Davies would carry information about the pre-inflationary evolution, opening an exciting window to explore such a remote era by looking at the CMB.

在暴胀场景依赖的诸多假设中，选择暴胀开始时扰动的初始状态尤为重要。暴胀的预言建立在: 对于宇宙微波背景中观测到的波长范围 $\lambda = 2\pi/k$ ，在暴胀起始阶段选择所谓的邦奇-戴维斯真空。这些波长远短于暴胀起始时的 (时空) 曲率半径 ($r_{\text{curv}} \approx 1/H$) ，邦奇-戴维斯真空对应这些短波长模式类闵氏真空涨落，加上次 Leading 修正。尽管这个前提初看很自然，但它假设这些模式在暴胀开始前的过去从未被激发过。鉴于我们对暴胀的起源以及暴胀之前的状况一无所知，这是一个很强的假设。例如，如果暴胀阶段之前发生过宇宙反弹，那么扰动进入暴胀时就不一定处于邦奇-戴维斯真空: 此时可观测模式可能已经进出过“视界”(更准确地说是曲率半径)，并且在这个过程中被激发。如果我们有一个暴胀前宇宙的场景，那么更合理的做法是从渐近过去很远之处开始演化，计算 (而非假设) 慢滚起始时的扰动状态 (不过这种方法仍然需要假设渐近过去很远之处扰动的初始状态)。偏离邦奇-戴维斯真空会携带暴胀前演化的信息，这为我们通过观测宇宙微波背景探索那个遥远的时代打开了一扇令人振奋的窗口。

This strategy was proposed and explored in LQC in [9, 11] and further analyzed from different perspectives [16, 85, 93, 208, 211, 214]. We start by listing the main steps in this program, emphasizing the choices and ambiguities at each step. These are (1) choice of an inflationary potential $V(\phi)$,(2) choice of an initial state for the background FLRW quantum spacetime Ψ_{hom} and for the perturbations Ψ_{pert} , and (3) evolution of the perturbations with one of the formalisms described above and the subsequent computation of observables of interest. We describe now these steps in some more detail.

该方法首先在圈量子宇宙学框架下由文献 [9, 11] 提出并研究，之后又从不同角度做了进一步分析 [16, 85, 93, 208, 211, 214]。我们首先列出该研究框架的主要步骤，重点说明每一步的选择和不确定性，分别是:(1) 选择暴胀势 $V(\phi)$, (2) 选择背景 FLRW 量子时空 Ψ_{hom} 和扰动 Ψ_{pert} 的初始状态，(3) 用上述形式化方法之一演化扰动，进而计算感兴趣的可观测物理量。下面我们对这些步骤做更详细的介绍。

1. Choice of the inflationary potential

1. 暴胀势的选择

At present, there is no compelling candidate for $V(\phi)$ within LQC. This is not surprising; one expects $V(\phi)$ to originate in the matter sector, which is introduced by hand in LQC rather than derived-although one cannot disregard the possibility that the inflaton field and its potential could have a purely gravitational origin [39,40].

目前，圈量子宇宙学 (LQC) 中还没有 $V(\phi)$ 的令人信服的候选方案。这并不意外，一般认为 $V(\phi)$ 源自物质部分，而在 LQC 中物质部分是人为引入的，并非推导得出——不过我们也不能排除暴胀子场及其势纯引力起源的可能性 [39,40]。

The strategy in LQC so far has been the same as in standard inflation: consider phenomenologically viable potentials and compare their results with observations. Several different forms of $V(\phi)$ have been analyzed in detail, including the simplest quadratic potential [11, 16] , the Starobinsky potential [66, 67, 211] , monodromy potential [186], and α -attractor potentials [183]. Of course, it is important to distinguish genuine LQC effects from those features arising from a concrete choice of $V(\phi)$.

迄今为止，LQC 采用的研究策略和标准暴胀理论一致: 研究唯象上可行的势，再将其结果和观测结果比对。目前已经详细分析了 $V(\phi)$ 的多种不同形式，包括最简单的二次势 [11, 16] 、斯塔罗宾斯基势 [66, 67, 211] 、单环绕势 [186]，以及 α 吸引子势 [183]。当然，将真正的 LQC 效应和由具体选择 $V(\phi)$ 带来的特征区分开是十分重要的。

As first discussed in [34,35] and further analyzed in [87,146,163], in the presence of a viable inflationary potential $V(\phi)$ the dynamics of the scalar field across the bounce of LQC can set up the appropriate conditions for inflation to start, quite generally if the kinetic energy of the scalar field dominates over its potential energy at the bounce. In this concrete sense, the attractor character of inflation in general relativity persists in LQC, if one assumes an appropriate potential for ϕ .

正如文献 [34,35] 首先讨论、并由文献 [87,146,163] 进一步分析的那样，当存在可行的暴胀势 $V(\phi)$ 时，标量场在 LQC 反弹过程的动力学可以普遍地为暴胀启动设定合适的条件，前提是反弹时标量场的动能主导其势能。从这个明确的意义来说，如果假设 ϕ 存在合适的势，那么广义相对论中暴胀的吸引子特性在 LQC 中依然成立。

2. Choice of the initial quantum state

2. 初始量子态的选择

To ensure a good semi-classical limit, the quantum background FLRW spacetime $\Psi_{\text{hom}}(a, \phi)$ is typically chosen to be a sharply peaked state, whose main features are captured by the effective equations of LQC. In this case, the freedom in the choice of solution is quite simple and in fact reduces to a one parameter freedom that dictates the length of the inflationary phase (as measured in number of e -folds N) . This point will be important when discussing predictions for the CMB.

为了得到良好的半经典极限，量子背景 FLRW 时空 $\Psi_{\text{hom}}(a, \phi)$ 通常被选为一个尖锐峰态，其主要特征可由 LQC 的有效方程描述。在这种情况下，解的选择自由度非常有限，实际上仅余一个单参数自由度，决定暴胀阶段的持续长度 (以 e 暴胀 N) 倍数计)。这一点在讨论 CMB 预言时十分重要。

Computing predictions for the CMB also requires choosing the state of perturbations, Ψ_{pert} , at some initial time, which can then be evolved across the pre-inflationary and inflationary phases. The predictions for the primordial power spectrum crucially depend on this choice. In the absence of a compelling way to specify the initial state, the theory would lack predictive power. The vacuum state is a natural choice, but, as is well known from quantum field theory in time-dependent spacetimes, the notion of vacuum is ambiguous except in very special circumstances. The strategy in LQC has been to add physical arguments to single out a choice and then compare the resulting predictions with observations; this may provide some confidence with the choice made or could rule it out. Thus, observations test not only the theory of LQC but also the arguments on how to fix the initial sate of perturbations.

计算 CMB 的预言还需要在某个初始时刻选定微扰的状态 Ψ_{pert} ，之后微扰会在 pre-暴胀和暴胀阶段演化。原初功率谱的预言关键依赖于这一选择。如果没有可靠的方式确定初始态，该理论就会缺乏预言能力。真空态是自然的选择，但如时变背景时空下量子场论的周知结论，除极特殊情况外，真空的定义是模糊的。LQC 中常用的研究思路是引入物理论证筛选出一个选择，再将得到的预言和观测对比；这可以帮助我们确认所选初始态的合理性，或是排除错误的选择。因此，观测不仅检验 LQC 理论本身，也检验确定微扰初始态的论证依据。

Multiple strategies have been proposed and explored in this regard. Perhaps the most conservative strategy is to fix the initial state of perturbations in the far past before the bounce. In addition of being a natural strategy in a bouncing universe, it has the advantage that far in the past and under mild assumptions, perturbations are far inside the curvature radius, and therefore all different notions of natural adiabatic vacua converge (for the same reason that everyone agrees on the vacuum state in labs on Earth, even though the universe is expanding) [12, 14, 16, 19, 20, 64, 65, 143, 168, 179, 211]. Notice that the same strategy is also used in alternatives to inflation, such as ekpyrosis and the matter bounce discussed in the subsections "Ekpyrosis in LQC" and "LQC Matter Bounce", since in these scenarios the primordial power spectrum is generated before the bounce and therefore one must specify the initial state far in the contracting branch.

学界已经针对这一问题提出并探索了多种策略。最保守的策略或许是在反弹前的遥远过去确定微扰的初始态。这不仅是反弹宇宙中自然的选择，还具备以下优势: 在温和假设下，遥远过去的微扰远在曲率半径之内，因此所有不同的自然绝热真空定义都会收敛 (这和地球实验室中大家对真空态的结论一致，即便宇宙在膨胀)[12, 14, 16, 19, 20, 64, 65, 143, 168, 179, 211]。需要注意的是，这一策略也被用在暴胀的替代模型中，例如“LQC 中的火劫论”和“LQC 物质反弹”小节讨论的火劫模型和物质反弹模型，因为在这些场景中原初功率谱是在反弹前产生的，因此必须在收缩分支的遥远过去指定初始态。

Another possibility is to use the bounce as a preferred time to specify the initial conditions [9, 10, 16, 85, 161, 211] . In this strategy, there is some ambiguity on the choice of initial state for a range of the smallest wavenumbers (longest wavelengths) we can observe in the CMB, since in scenarios of phenomenological interest these wavelengths are greater than the curvature radius at the bounce. Different proposals for a preferred vacuum state at (or near) the bounce have been considered so far using two arguments, namely, the extrapolation of the adiabatic series [9,11,16] and minimization conditions for the expectation value of the energy-momentum tensor [17, 161]. Interestingly, although prescriptions using either of these two strategies differ significantly in their details, they all produce very similar observational predictions [16, 161], which are also quite similar to the results obtained when specifying adiabatic initial conditions far in the past of the bounce.

另一种可能是以反弹时刻作为优先时刻来指定初始条件 [9, 10, 16, 85, 161, 211] 。在该策略下，对于我们能在 CMB 中观测到的最小波数 (最长波长) 范围，初始态的选择存在一定歧义，因为在唯象上有意义的场景中，这些波长大于反弹时刻的曲率半径。迄今为止，学界已经基于两个论证考虑了在反弹处 (或反弹附近) 选择优先真空态的不同方案，即绝热级数外推 [9,11,16] 和能量动量张量期望的极小化条件 [17, 161]。有意思的是，尽管这两种策略给出的方案在细节上差异很大，它们的观测预言都非常接近 [16, 161]，也和在反弹遥远过去指定绝热初始条件得到的结果十分相似。

A third strategy that has been proposed to fix the state of perturbations is to impose conditions on Ψ_{pert} at more than one instant, so these conditions are nonlocal in time-e.g., conditions the state must satisfy both at the bounce and the end of inflation [28, 29, 100] or during an interval of the pre-inflationary evolution [93, 160]. The two existing proposals based on this third strategy do predict a primordial power spectra substantially different from the ones obtained with either of the other two strategies summarized above; we discuss this further at the end of the subsection ”Inflation in the Dressed Metric and Hybrid Approaches”.

第三种已被提出的确定微扰态的策略是在多个时刻对 Ψ_{pert} 施加条件，因此这些条件是时间非定域的——例如，要求态同时满足反弹处和暴胀结束处 [28, 29, 100] 的条件，或是在 pre-暴胀演化的某一区间内满足条件 [93, 160]。基于该第三种策略的两个已有方案确实预言了和上述另外两种策略截然不同的原初功率谱；我们会在“带度规穿衣的暴胀与混合方法”小节的末尾进一步讨论这一点。

3. Computation of predictions in the different frameworks

3. 不同框架下的预言计算

In the remainder of this section, we provide a summary of the results obtained using the different frameworks described in the section ”Cosmological Perturbations in LQC”, describing the predictions for observables of interest in primordial cosmology, including the amplitude of scalar and tensor primordial perturbations and their spectral indices and runnings. Non-Gaussianity and primordial anisotropies are discussed in the section ”Extensions”. Particular attention has also been paid to the so-called large-scale anomalies in the CMB [23].

在本节剩余部分，我们将总结使用“圈量子宇宙学中的宇宙扰动”一节介绍的不同框架得到的结果，阐述原初宇宙学中感兴趣的可观测效应的预言，包括原初标量、张量扰动的振幅，以及它们的谱指数和跑动。非高斯性和原初各向异性将在“扩展”一节讨论。学界还特别关注了宇宙微波背景 (CMB) 中所谓的大尺度异常 [23]。

Inflation in the Dressed Metric and Hybrid Approaches

修正度规方法与混合方法中的暴胀

Since the dressed metric and hybrid quantization approaches give very similar predictions for the primordial scalar and tensor power spectra in inflation, we discuss them together here; see [145] for a detailed comparison between the two approaches.

由于修正度规和混合量子化方法对暴胀中原初标量和张量功率谱的预测非常相似，我们在此将它们放在一起讨论；两种方法的详细比较参见文献 [145]。

Before describing the results for the power spectrum, it is informative to acquire first an intuitive understanding of the type of modifications LQC can cause, relative to standard inflation, and their physical origin.

在描述功率谱的结果之前，我们首先需要了解圈量子宇宙学 (LQC) 相比标准暴胀会带来何种修改、这些修改的物理起源，这对后续讨论十分重要。

The argument is simplest for tensor modes χ_k (with χ the analog for tensor perturbations of the Mukhanov-Sasaki variable v) since $\mathcal{U}_{\text{tensor}} = 0$; the equation of motion is slightly more complicated for scalar perturbations, but the general argument remains the same. Since $\mathcal{U}_{\text{tensor}} = 0$, and recalling that the Ricci curvature of the dressed metric is $R = 6\tilde{a}''/\tilde{a}^3$, the equation of motion for tensor modes is

对于张量模式 χ_k ，推导过程最为简单 (其中 χ 是张量扰动对应于穆哈诺夫-佐佐木变量 v 的类比)，因为 $\mathcal{U}_{\text{tensor}} = 0$；标量扰动的运动方程稍复杂，但整体推导逻辑是一致的。由于 $\mathcal{U}_{\text{tensor}} = 0$，且修正度规的里奇曲率为 $R = 6\tilde{a}''/\tilde{a}^3$ ，因此张量模式的运动方程为

$$\chi_k''(\eta) + \tilde{a}^2\left(\frac{k^2}{\tilde{a}^2} - \frac{1}{6}R\right)\chi_k(\eta) = 0. \tag{20}$$

This equation shows that the evolution of χ_k is dictated by a competition between the physical wavenumber squared of the Fourier mode k and the Ricci scalar curvature. If $(k/\tilde{a})^2 \gg R, R$ can be neglected and then $\chi_k''(\eta) + k^2\chi_k(\eta) = 0$, which is the equation we would have found in Minkowski spacetime and whose solutions are linear combinations of positive and negative frequency modes $e^{\pm ik\eta}$. It is known from quantum field theory in Minkowski spacetime that this simple evolution does not create particles or excite the vacuum state. Restated in terms of wavelengths, the Fourier modes whose wavelength are much smaller than the ”radius of curvature” $r_{\text{curv}} = \sqrt{6/R}$, behave as in flat spacetime, and vacuum fluctuations remain unexcited.

该方程表明，χ_k 的演化由傅里叶模 k 的物理波数平方与里奇标量曲率之间的竞争决定。若 $(k/\tilde{a})^2 \gg R, R$ 可忽略，此时方程变为 $\chi_k''(\eta) + k^2\chi_k(\eta) = 0$ ，这就是我们在闵氏时空中得到的方程，其解是正负频率模 $e^{\pm ik\eta}$ 的线性组合。根据闵氏时空量子场论我们知道，这种简单演化不会产生粒子也不会激发真空态。换用波长表述即: 波长远小于“曲率半径”$r_{\text{curv}} = \sqrt{6/R}$ 的傅里叶模，其行为与平直时空一致，真空涨落保持未激发状态。

On the contrary, when the physical wavenumber squared becomes comparable to the curvature $(k/\tilde{a})^2 \sim R$, the effective frequency of oscillation of χ_k becomes time-dependent and complex exponential $e^{\pm ik\eta}$ are no longer solutions; this is the regime where perturbations are affected by the curvature of the background spacetime.

反之，当物理波数平方与曲率 $(k/\tilde{a})^2 \sim R$ 大小相当时，χ_k 的有效振荡频率随时间变化，复指数 $e^{\pm ik\eta}$ 不再是方程的解；在这个区域，背景时空的曲率会对扰动产生影响。

To get an idea of when primordial perturbations may become excited, it is sufficient to compare the evolution of their physical wavelength $\lambda(t)$ with the curvature radius $r_{\text{curv}}(t)$. An example of this is shown in Fig. 1, for the case of a quadratic potential $V(\phi) = \frac{1}{2}m^2\phi^2$ and a choice of initial conditions that produces a total of $68e$ -folds of inflation. The red line shows the radius of curvature from some time before the bounce until the end of inflation, while the gray shadowed band indicates the range of wavelengths observed in the CMB. For this concrete evolution, the longest wavelengths observed today in the CMB become larger than the curvature radius near the time of the bounce-during this interval, perturbations with these wavelengths are affected by the background spacetime curvature, and as a result, when inflation starts after the bounce, these wavelengths are already in an excited state compared to the Bunch-Davies vacuum. On the other hand, the shortest wavelengths observed in the CMB today remain much smaller than r_{curv} during the entire Planck era and only become comparable to r_{curv} at much later times during inflation, so these wavelengths will reach the onset of inflation in the vacuum state.

想要了解原初扰动何时会被激发，只需将其物理波长 $\lambda(t)$ 与曲率半径 $r_{\text{curv}}(t)$ 进行比较。图 1 给出了一个二次势 $V(\phi)=\frac{1}{2}m^2\phi^2$ 的例子，其初始条件总共产生了 $68e$ 个 e 折暴胀。红线展示了从反弹前一段时间到暴胀结束的曲率半径，灰色阴影区表示宇宙微波背景 (CMB) 中观测到的波长范围。对于这个具体演化过程，如今 CMB 中观测到的最长波长在反弹时刻附近就已经大于曲率半径——在这段时间里，这些波长的扰动会受到背景时空曲率的影响，因此当反弹后暴胀开始时，与邦奇-戴维斯真空相比，这些波长已经处于激发态。另一方面，如今 CMB 中观测到的最短波长在整个普朗克时代都远小于 r_{curv}，直到暴胀后期才变得与 r_{curv} 相当，因此这些波长在暴胀开始时仍处于真空态。

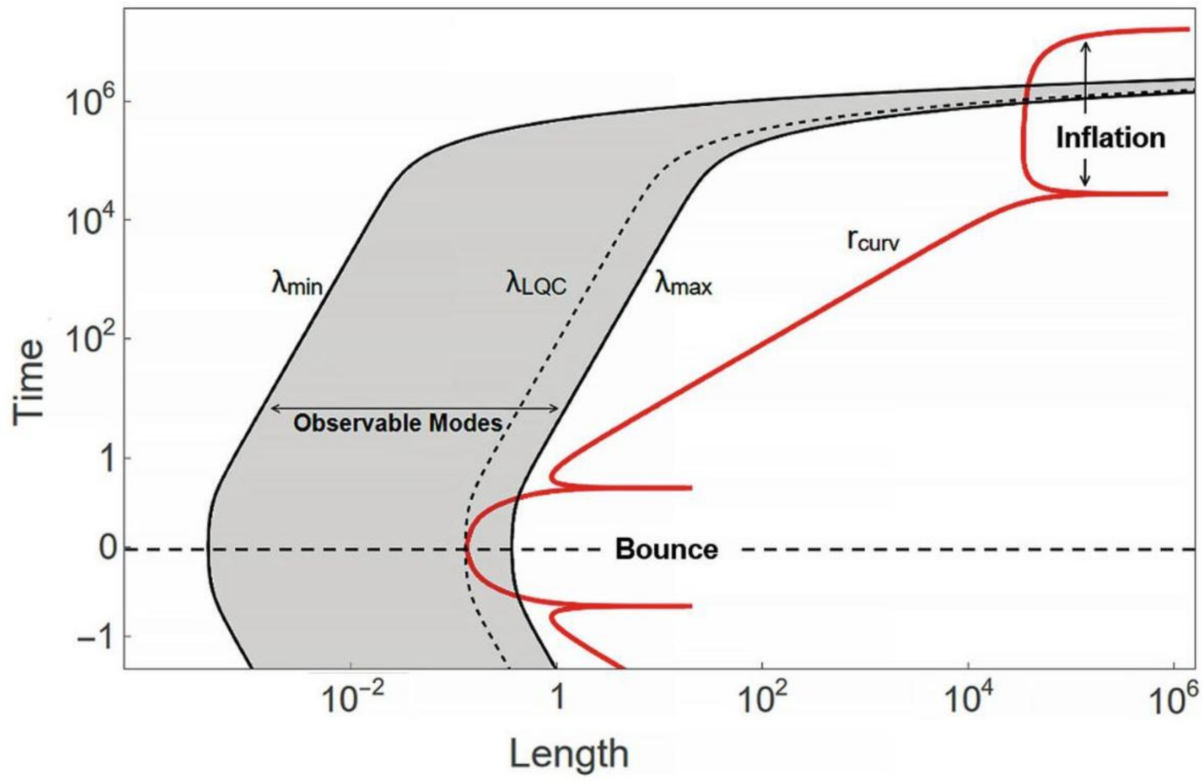

Fig. 1 This plot shows the LQC bounce followed by a phase of inflation. The "curvature scale" $r_{\text{curv}}(t)$ is shown as a red line, while the gray region corresponds to the range of wavelengths observed in the CMB today. (Note that $r_{\text{curv}}(t)$ diverges when the Ricci scalar changes sign; this happens just before and after the bounce, as well as before and after inflation.) In this scenario, the longest-wavelength modes observed in the CMB today were comparable to or larger than the curvature scale near the bounce. Consequently, these modes were excited at that time and can provide an observational window to the Planck era

图 1 该图展示了圈量子宇宙学 (LQC) 的反弹后接暴胀阶段的过程。“曲率尺度” $r_{\text{curv}}(t)$ 以红线显示，灰色区域对应如今宇宙微波背景 (CMB) 中观测到的波长范围。(注: 当里奇标量改变符号时 $r_{\text{curv}}(t)$ 发散；该现象发生在反弹前后，以及暴胀前后。) 在该图景中，如今 CMB 观测到的最长波长模式，在反弹附近与曲率尺度相当或更大。因此这些模式在当时被激发，可为普朗克时代提供观测窗口

The division between wavelengths that "feel" the geometry during the bounce, and those that do not, is determined by the value of the curvature radius at the bounce $r_{\text{curv}}(t_B)$: this is the physical scale that LQC introduces in the physics of primordial perturbations. Wavelengths satisfying $\lambda(t_B) \gtrsim r_{\text{curv}}(t_B)$ will carry some information about LQC. In terms of comoving wavenumbers, the modes with $k \lesssim k_{\mathrm{LQC}}$, where $k_{\mathrm{LQC}} \equiv 1/r_{\text{curv}}(t_B)$, are the "messengers" from the Planck era.

反弹过程中“能感知”几何的波长与无法感知的波长之间的分界由反弹处的曲率半径 $r_{\text{curv}}(t_B)$ 决定: 这是 LQC 引入原初扰动物理中的特征尺度。满足 $\lambda(t_B) \gtrsim r_{\text{curv}}(t_B)$ 的波长会携带关于 LQC 的信息。对共动波数而言，满足 $k \lesssim k_{\mathrm{LQC}}$ (其中 $k_{\mathrm{LQC}} \equiv 1/r_{\text{curv}}(t_B)$) 的模式就是来自普朗克时代的“信使”

As mentioned above, this discussion can be extended to scalar modes with essentially the same result, as the potential $\tilde{\mathcal{U}}$ does not substantially change the argument.

如上所述，该讨论可以推广到标量模式，结果基本不变，因为势 $\tilde{\mathcal{U}}$ 不会从根本上改变这一论证

Leaving aside qualitative arguments, it is possible to solve the dynamics explicitly numerically (for analytic results, see [151, 208, 211-213, 216]) and calculate the observables of interest, for example, the power spectra at the end of inflation.

撇开定性论证不谈，我们可以显式数值求解动力学 (解析结果见文献 [151, 208, 211-213, 216])，并计算感兴趣的观测量，例如暴胀末期的功率谱

As an example, Fig. 2 shows the scalar power spectrum calculated for a particular scenario and choice of initial conditions. Specifically, the inflationary potential chosen here is $V(\phi) = \frac{1}{2}m^2\phi^2$ with $m = 1.3 \times 10^{-6}$ being used in this plot (note that the qualitative result for LQC effects on the power spectrum is very similar for other inflationary potentials) and initial conditions for the background such that there are about 4 e -folds from the bounce to the onset of inflation, and approximately $64e$ -folds of inflation, while the initial conditions for the perturbations are obtained using the so-called preferred instantaneous vacuum [17] defined at $t_i = -50{,}000 t_{\mathrm{Pl}}$ before the bounce (this is a vacuum state of fourth adiabatic order and the choice of t_i does not affect the results so long as it is far before the bounce). This plot shows that, as expected, LQC effects appear on infrared scales $k < k_{\mathrm{LQC}}$, while predictions for the modes $k > k_{\mathrm{LQC}}$ are unchanged from the standard inflationary scenario. In this scenario, modes modified by LQC can be observed in the CMB, although only in the most infrared sector. Concretely, LQC modifications appear for $\ell \lesssim 30$ in the angular power spectrum C_ℓ for this choice of parameters. Results for tensor modes are similar, although with a smaller amplitude [17].

例如，图 2 展示了针对特定情景与初始条件选择计算得到的标量功率谱。具体而言，此处选取的暴胀势为 $V(\phi) = \frac{1}{2}m^2\phi^2$ ，本图采用了 $m = 1.3 \times 10^{-6}$ (请注意，对于其他暴胀势，圈量子宇宙学效应影响功率谱的定性结果非常相似)，背景初始条件使得从 bounce 到暴胀开始约有 4 个 e 暴胀倍数，总暴胀倍数约为 $64e$ ，而扰动的初始条件则通过定义在 bounce 前 $t_i = -50{,}000 t_{\mathrm{Pl}}$ 处的所谓优先瞬时真空得到 [17]，这是一种四阶绝热真空，只要 t_i 远在 bounce 之前，其选取就不会影响结果。该图表明，不出所料，LQC 效应出现在红外标度 $k < k_{\mathrm{LQC}}$ ，而模式 $k > k_{\mathrm{LQC}}$ 的预测与标准暴胀情景一致。在该情景下，虽然仅存在于极红外区域，LQC 修正的模式仍可以在宇宙微波背景中被观测到。具体来说，对于该参数选择，LQC 修正出现在角功率谱 C_ℓ 的 $\ell \lesssim 30$ 处。张量模式的结果与此类似，只是振幅更小 [17]。

In summary, LQC introduces a new physical scale k_{LQC} in the physics of primordial perturbations. The scale invariance of the power spectrum generated by inflation is modified for modes $k < k_{\mathrm{LQC}}$: the spectral indices of both scalar and tensor perturbations become more negative, or, equivalently, the running of the two spectral indices increases. Note that the modifications that LQC introduces for scalar and tensor modes are very similar, so the tensor-to-scalar ratio r remains the same as in standard inflation. This implies that the consistency relation $r = -8n_t$, where n_t is the tensor spectral index, valid in standard inflation, is not satisfied in LQC for modes $k < k_{\mathrm{LQC}}$ for which $r < -8n_t$ instead [11, 16] .

综上所述, 圈量子宇宙学 (LQC) 在原初扰动物理中引入了一个新的物理标度 k_{LQC} 。对于模 $k < k_{\mathrm{LQC}}$ ，暴胀产生的功率谱的标度不变性发生了改变: 标量扰动和张量扰动的谱指数都会变得更负，换句话说，两个谱指数的跑动增大。值得注意的是，LQC 对标量模和张量模的修正非常相似，因此张标比 r 仍与标准暴胀中的结果一致。这意味着，标准暴胀中成立的、其中 n_t 为张量谱指数的一致性关系 $r = -8n_t$ ，在 LQC 中对满足 $r < -8n_t$ 而非 [11, 16] 的模 $k < k_{\mathrm{LQC}}$ 不再成立。

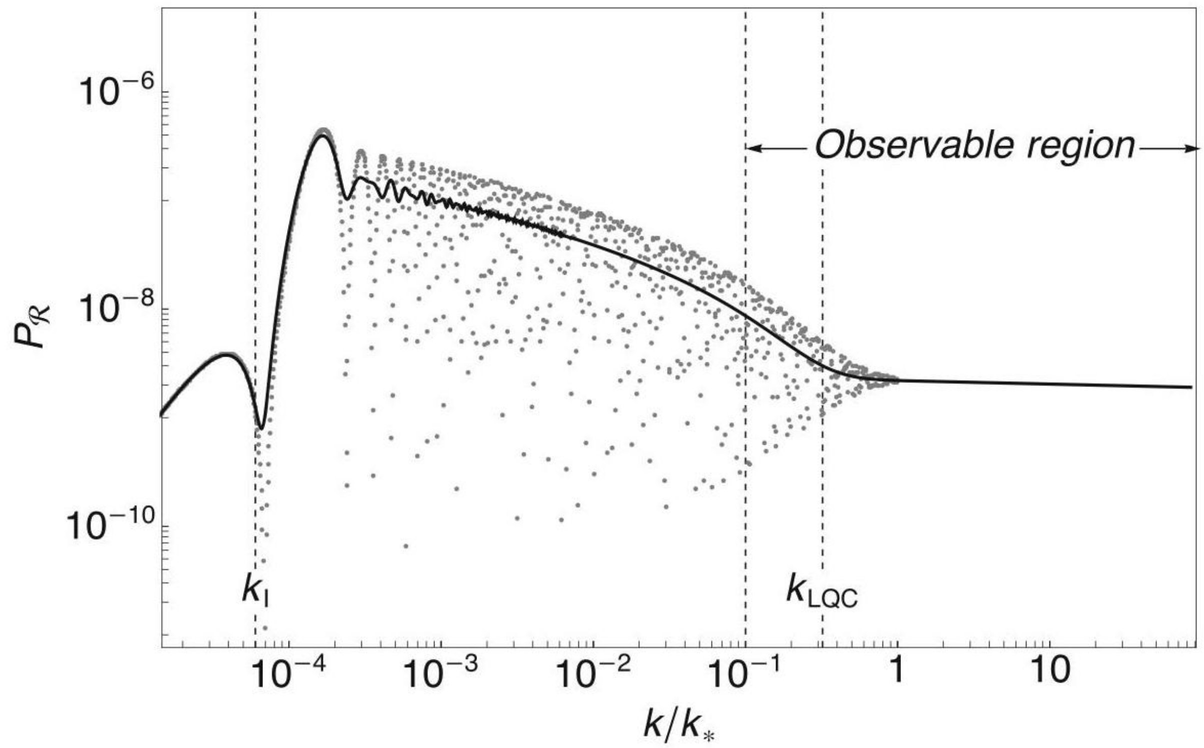

Fig. 2 Scalar power spectrum in LQC versus wavenumber k/k_* , where k_* is a reference comoving scale corresponding to 0.002Mpc^{-1} today. The LQC characteristic scale k_{LQC} is denoted by a vertical line, and the gray dots show the scalar power spectrum, computed for a discrete set of wavenumbers. The power spectrum oscillates rapidly and its average is shown in black. It is almost scale-invariant and in agreement with the predictions of standard inflationary with Bunch-Davies initial conditions for $k > k_{\mathrm{LQC}}$. On the contrary, for $k < k_{\mathrm{LQC}}$ the near scale invariance is broken due to LQC effects that excited these modes during the Planck era. The plot also shows, for a certain choice of parameters, the region of Fourier modes that are observable in the CMB as well as k_I , the most infrared mode that exits the horizon during the inflationary phase of the universe [16]

图 2 LQC 中标量功率谱随波数 k/k_* 的变化，其中 k_* 是对应如今 0.002Mpc^{-1} 的共动参考标度。LQC 特征标度 k_{LQC} 由竖线标记，灰点展示了对一组离散波数计算得到的标量功率谱。功率谱存在快速振荡，其平均值以黑线标出。对于 $k > k_{\text{LQC}}$，功率谱几乎满足标度不变性，与 Bunch-Davies 初始条件下标准暴胀的预测一致。相反，对于 $k < k_{\text{LQC}}$，由于 LQC 效应在普朗克时期激发了这些模，近标度不变性被破坏。图中还展示了某一参数选择下，CMB 中可观测的傅里叶模区域，以及 k_I ——即在宇宙暴胀阶段出视界的最红外模 [16]。

Similar results have been obtained using other choices for the quantum state of perturbations, specified at or before the bounce [16], although some quantities like the slope of the power spectrum in the intermediate region change slightly [149]. In addition, choices motivated by arguments that are non-local in time [28,93,100,160] produce LQC corrections that reduce the primordial power spectrum, rather than enhance it. The sign of the spectral indices and runnings is, therefore, reversed if one used one of these states based on the non-local conditions, and the consistency relation is modified in the inverse manner to $r > -8n_t$ for infrared wavenumbers [28, 93, 160] .

即使改用反弹时刻或反弹之前指定的其他扰动量子态,也能得到类似的结果 [16],仅中间区域功率谱斜率这类部分量会发生微小变化 [149]。此外，由时间非局域论证得到的量子态选择 [28,93,100,160] 会产生 LQC 修正，这类修正会压低而非抬高原初功率谱。因此，若采用这类基于非局域条件的量子态，谱指数和跑动的符号都会反转，一致性关系的修正方式也与红外波数 [28, 93, 160] 对应的 $r > -8n_t$ 相反。

Large-scale Anomalies in the CMB

宇宙微波背景中的大尺度异常

Since LQC modifies the primordial power spectrum at infrared scales, this raises the question of whether LQC can provide a natural mechanism to account for the anomalous features observed in the correlation of temperature anisotropies at large angular separations; these are called large-scale anomalies in the CMB (see, e.g., [182] for a summary). These anomalies include (i) the absence of two-point correlations at large scales, also known as power suppression; (ii) a hemispherical or dipolar asymmetry; (iii) a bias for odd-parity correlations; and (iv) a preference of data for a value of the lensing amplitude A_L larger than one [6]. Most of these signals have been detected in data from the satellites WMAP and Planck, and some have been noticed even in data from COBE; this rules out an origin from instrumental noise or residual systematics. There is an agreement that these signals are real features in the CMB. The discussion is instead whether the observed signals require new physics. The significance of these features has been quantified by using the p -value [1] that measures the probability of observing each of these features in a standard ΛCDM universe. The three anomalies separately have similar p -values, of the order of a fraction of percent [2, 23] . These p -values are small, but not sufficiently small to close the discussion about the need of new physics. It is important to notice though that these are the p -values of each of the anomalies separately. Their collective p -value must be smaller, so their collective significance is higher.

由于圈量子宇宙学 (LQC) 修改了红外尺度上的原初功率谱，这便引出一个问题:LQC 能否提供一种自然机制，解释大角尺度温度各向异性关联中观测到的反常特征；这些特征被称为宇宙微波背景 (CMB) 的大尺度异常 (综述可参见文献 [182])。这些异常包括:(i) 大尺度上不存在两点关联，即功率抑制；(ii) 半球不对称或偶极不对称；(iii) 奇宇称关联偏好；(iv) 观测数据倾向于透镜振幅 A_L 大于 1[6]。大多数信号已在 WMAP 和普朗克卫星的数据中被检测到，部分信号甚至在 COBE 的数据中就已被发现；这排除了这些信号源自仪器噪声或残余系统误差的可能。学界一致认为这些信号是 CMB 中真实存在的特征，争议点在于观测到的这些信号是否需要新物理来解释。学界已经使用 p 值量化了这些特征的显著性，该值衡量了在标准 ΛCDM 宇宙中观测到每个特征的概率。三个异常单独存在时具有相近的 p 值，约为千分之几到百分之几 [2, 23] 。这些 p 值很小，但还没有小到可以盖棺定论，说明仍无法排除对新物理的需求。但需要注意，这些只是每个异常单独存在时的 p 值。它们的整体 p 值必然更小，因此整体显著性更高。

Within LQC, there have been two different proposals to account for these anomalies, which we summarize in the following.

在 LQC 框架内，目前存在两种解释这些异常的不同方案，我们在此做总结。

The first proposal points out that the non-Gaussianity generated by the LQC bounce can make the observed features much more likely than in standard Λ CDM [7, 14]. The underlying idea is based on the ”non-Gaussian modulation” mechanism [5, 7, 89, 131, 180, 181] where correlations between CMB modes and super-horizon modes can bias the observed power spectrum, making certain features to be more likely. An important challenge for non-Gaussian modulation is to generate an appropriate form of non-Gaussianity that is small when the three wavenumbers lie within the observable window, in order to respect observational constraints, but that grows significantly when one of the modes is super-horizon. It is interesting that these are precisely the properties of the non-Gaussianity generated by the LQC bounce [12] (for details, see the subsection ”Non-Gaussianity”), in such a way that the resulting non-Gaussian modulation due to LQC can alleviate the four large-scale anomalies mentioned above [13-15]. The way this alleviation happens is statistical: LQC does not predict that such anomalies must be necessarily present in our universe, but rather makes their p -values significantly larger than in ΛCDM , so the reason why these features were called anomalies in the first place disappears. (The analysis also makes predictions for tensor modes which can be contrasted with observations if these modes are eventually measured.)

第一种方案指出，LQC 反弹产生的非高斯性可以使观测到的这些特征比标准 Λ ΛCDM 模型中更有可能存在 [7, 14]。其核心思想基于“非高斯调制”机制 [5, 7, 89, 131, 180, 181] ，即 CMB 模式与超视界模式的关联可以偏差观测功率谱，让特定特征更有可能出现。非高斯调制面临的一个重要挑战是，需要生成形式合适的非高斯性: 当三个波数都处于可观测窗口内时非高斯性很小，以符合观测约束；当其中一个模式处于超视界时非高斯性显著增长。有趣的是，LQC 反弹产生的非高斯性恰好具备这些性质 [12](详情参见“非高斯性”小节)，因此 LQC 带来的非高斯调制可以缓解上述四大类大尺度异常 [13-15]。这种缓解是统计层面的:LQC 并不预言我们的宇宙必然存在这类异常，而是让它们的 p 值比 ΛCDM 中的结果大得多，因此这些特征最初被称为异常的原因也就不复存在了。(该分析还对张量模做出了预言，如果这些张量模最终被观测到，就可以和观测结果比对。)

The main strength of this idea is that it can account for several anomalies, of very different nature, like the power suppression and the dipolar asymmetry, while the main limitation is its inability to incorporate the rapid oscillations of the LQC bispectrum. Since these oscillations may reduce the effects of non-Gaussianity in the observed CMB, the results of [14] must be understood as an upper bound for the significance of the anomalies within LQC, rather than a sharp prediction. We comment on other possible observational signatures of this scenario for non-Gaussianities in the subsection ”Non-Gaussianity”.

该方案的主要优势是它可以解释多种性质完全不同的异常，比如功率抑制和偶极不对称；主要局限则是它无法纳入 LQC 双谱的快速振荡。由于这些振荡可能会削弱非高斯性在观测 CMB 中的效应，因此文献 [14] 的结果应当被理解为 LQC 框架下异常显著性的上限，而非精确预言。我们会在“非高斯性”小节讨论该非高斯性场景下其他可能的观测信号。

The other avenue explored within LQC to account for some of the anomalies is based on ”initial” conditions for the perturbations that are non-local in time. Specifically, for the quantum state singled out by demanding that the power spectrum at the end of inflation does not oscillate in k , the scalar power spectrum is suppressed in the infrared part of the visible window [93]. A similar result was obtained by using a quantum state for scalar perturbations obtained by combining a quantum generalization of Penrose’s null Weyl curvature hypothesis with the demand that the state has minimum uncertainty in the curvature perturbations (and therefore maximum uncertainty in the canonically conjugated momentum) at the end of inflation [28,29]. Further, the authors of [30,31] noticed that this suppression can also alleviate the observed anomaly in the lensing parameter A_L , by making the observed value compatible with 1 within 1σ ; this analysis comes with new predictions of a larger optical depth and power suppression for the B -mode polarization.

LQC 中为解释部分异常探究的另一途径，基于对微扰而言时间非定域的「初始」条件。具体而言，对于要求暴胀末期功率谱不在 k 中振荡而选出的量子态，可见窗口的红外部分的标量功率谱会被压低 [93]。结合彭罗斯零外尔曲率假设的量子推广，以及要求该量子态在暴胀末期曲率扰动中满足最小不确定度 (因此正则共轭动量满足最大不确定度)，得到的标量扰动量子态也给出了相似结果 [28,29]。此外，文献 [30,31] 的作者发现，这种压低还能缓解观测到的透镜参数 A_L 异常，使得观测值在 1σ 范围内与 1 相容；该分析还给出了新预言: 更大的光深以及 B 模偏振的功率压低。

A First Look at Quantum Ambiguities: Inflation in Modified LQC

量子不确定性初探: 修正圈量子宇宙学中的暴胀

An outstanding issue in LQC is the connection between it and the full theory of LQG. The starting point for LQC is to reduce the classical Hamiltonian from infinitely many to a few gravitational degrees of freedom by imposing homogeneity (isotropy further reduces the system to a single degree of freedom), and the reduced system is quantized using the techniques of LQG. However, the processes of symmetry reduction and quantization do not commute in general, and it is important to understand how well the physics of the cosmological sector of full LQG is captured by LQC. In the past decade or so, this important issue has been extensively studied by both bottom-up and top-down approaches; from these studies, an important conclusion is emerging: *LQC* and its major predictions are robust. In particular, the big bang singularity is resolved in all the models studied so far, and predictions for cosmological perturbations are consistent with current cosmological observations for a wide variety of initial conditions.

圈量子宇宙学 (LQC) 的一个突出问题是它与完整圈量子引力 (LQG) 理论之间的关联。LQC 的出发点是通过施加均匀性条件，将经典哈密顿量从无穷多个引力自由度约化为少数几个 (各向同性进一步将系统约化为单个自由度)，再利用 LQG 方法对约化系统量子化。但一般来说，对称性约化过程与量子化过程不对易，因此弄清完整 LQG 宇宙学区域的物理内容被 LQC 捕捉的程度十分重要。过去十余年里，该重要问题得到了自下而上与自上而下两类方法的广泛研究；这些研究逐渐得出一个重要结论: *LQC* 及其核心预言都是稳健的。尤其是，大爆炸奇点在目前所有研究模型中都得到了解决，且针对宇宙学微扰的预言在大范围初始条件下都与现有宇宙学观测一致。

We will review results from the top-down approach in the section ”Beyond LQC”, while we focus here on the bottom-up approach. In this setting, symmetries are still imposed before quantization, but with the observation that the Hamiltonian constraint can be written in different equivalent forms at the classical level. In one commonly used form, the constraint contains two terms often called ”Euclidean” and ”Lorentzian,” respectively (since only the first term appears in the Hamiltonian constraint for Euclidean gravity). For the spatially flat FLRW universe, these two parts are proportional in the classical theory, and this simplification is typically used in LQC to reduce the total Hamiltonian constraint to a single Euclidean term (although with a different factor of proportionality). Since the Euclidean and Lorentzian terms are usually regularized differently in LQG, this motivates treating the Lorentzian term independently by applying Thiemann’s regularization [192] of the full theory of LQG to LQC [209], with the result that the wave function is described by a fourth-order difference equation [91, 209], rather than the second-order difference equation appearing in standard LQC. For sharply peaked states, the resulting quantum dynamics are well described by effective Friedman-Raychaudhuri equations [146-148].

我们会在“LQC 之外”一节回顾自上而下方法的结果，本文这里聚焦自下而上方法。在该框架中，依然是先施加对称性再量子化，但我们注意到哈密顿约束在经典层面可以写成多种等价形式。在一种常用形式中，约束包含两个项，通常分别称为“欧几里得项”和“洛伦兹项”(因为欧几里得引力的哈密顿约束中仅包含第一项)。对于空间平坦的 FLRW 宇宙，这两部分在经典理论中是成正比的，LQC 通常利用这一简化将总哈密顿约束约化为单个欧几里得项 (尽管比例系数不同)。由于欧几里得项和洛伦兹项在 LQG 中通常的正则化方式不同，这推动了研究者将洛伦兹项独立处理，把完整 LQG 的 Thiemann 正则化 [192] 应用到 LQC[209] 中，最终得到波函数由四阶差分方程描述 [91, 209]，而非标准 LQC 中的二阶差分方程。对于尖峰态，所得量子动力学可以用有效弗里德曼-雷乔杜里方程很好地描述 [146-148]。

This version of LQC is often called mLQC-I, with ”m” for modified [147]. Note that another modified version of LQC, called mLQC-II, is obtained by imposing that the spin-connection vanishes before quantizing the Euclidean and Lorentzian terms separately [209]. It has since been shown that the physics of mLQC-II is very similar to standard LQC [143, 146-150], and therefore we will focus on mLQC-I here.

这个版本的 LQC 通常被称为 mLQC-I，其中“m”代表修正 [147]。注意还存在另一个修正 LQC 版本，称为 mLQC-II，它是通过在分别量子化欧几里得项和洛伦兹项之前施加自旋联络为零的条件得到的 [209]。已有研究表明，mLQC-II 的物理性质与标准 LQC 非常相似 [143, 146-150]，因此我们这里聚焦 mLQC-I。

Comparing mLQC-I with LQC, one finds that the big bang singularity is still replaced by a quantum bounce, but the critical energy density the bounce occurs at in mLQC-I is smaller by a factor of $4\left(1+\gamma^2\right)$. More significant differences arise comparing the pre-bounce eras: in standard LQC the classical Einstein dynamics are recovered far from the bounce, both to the past and future, while in mLQC-I this is only true for one side of the bounce. If we assume the post-bounce era to have a good classical limit to match our expanding universe, then in mLQC-I the pre-bounce universe rapidly approaches a de Sitter phase, with a Planckian effective cosmological constant. Despite the pre-bounce differences, the post-bounce dynamics of standard LQC and mLQC-I are essentially identical [146-148].

对比 mLQC-I 与 LQC 可以发现，大爆炸奇点依然会被量子反弹替代，但 mLQC-I 中反弹发生时的临界能量密度比标准 LQC 小 $4\left(1+\gamma^2\right)$ 倍。反弹前时代的差异更显著: 在标准 LQC 中，无论过去还是未来，远离反弹处都能回到经典爱因斯坦动力学，而在 mLQC-I 中只有反弹一侧满足这一点。如果我们假设反弹后的时代存在良好经典极限来匹配我们的膨胀宇宙，那么 mLQC-I 中反弹前的宇宙会迅速趋近德西特相，有效宇宙学常数为普朗克量级。尽管反弹前存在差异，标准 LQC 与 mLQC-I 反弹后的动力学基本一致 [146-148]。

In mLQC-I, an inflationary phase generally occurs, assuming an inflaton with a suitable potential [148]. If the inflaton is kinetic-dominated at the bounce, $\dot{\phi}_B^2/2 \gg V(\phi_B)$, then inflation always happens at $t_i \simeq 10^4 - 10^6 t_{\mathrm{Pl}}$. On the other hand, if the scalar field is initially dominated by its potential energy at the bounce, inflation does not always happen; this is true not only in mLQC-I [146-148] but also in standard LQC [66, 67]. Note that kinetic-dominated initial states can be expected to arise quite generally, given the Hubble anti-friction term in the Klein-Gordon equation for a contracting universe which will significantly increase $\ddot{\phi}$.

在 mLQC-I 中，只要暴胀子具有合适的势，通常就会产生暴胀相 [148]。如果反弹时暴胀子以动能为主，$\dot{\phi}_B^2/2 \gg V(\phi_B)$，那么总会在 $t_i \simeq 10^4 - 10^6 t_{\mathrm{Pl}}$ 发生暴胀。反之，如果反弹时标量场初始以势能为主，就不一定会发生暴胀；这一结论不仅对 mLQC-I 成立 [146-148]，对标准 LQC 同样成立 [66, 67]。注意到，动能主导的初态其实可以很自然地产生: 收缩宇宙的克莱因-戈登方程中存在哈勃反摩擦项，该项会显著增大 $\ddot{\phi}$。

The evolution of the homogeneous universe is simple and universal for initial states dominated by kinetic energy at the LQC bounce. In terms of the effective equation of state of the inflaton, $w(\phi) \equiv \left[\dot{\phi}^2 - 2V(\phi)\right] / \left[\dot{\phi}^2 + 2V(\phi)\right]$, it has the value $w(\phi) \simeq 1$ during a long kinetic-dominated era (of $\Delta t \simeq 10^5 t_{\mathrm{Pl}}$), and the potential energy remains nearly constant. When $t - t_B \simeq 10^5 t_{\mathrm{Pl}}$, the kinetic energy suddenly drops and $w(\phi) \to -1$, signaling the start of inflation when the potential energy takes over. Therefore, there are three phases before reheating: the bounce, the kinetic transition, and then inflation [146-148]. Note that similar behavior has been also found in standard LQC for kinetic-dominated initial conditions for the inflaton at the bounce [211, 214] and is true for a wide range of inflationary potentials [44, 183- 187].

在 LQC 反弹处，对于动能主导的初态，均匀宇宙的演化简单且普适。就暴胀子的有效物态方程 $w(\phi) \equiv \left[\dot{\phi}^2 - 2V(\phi)\right] / \left[\dot{\phi}^2 + 2V(\phi)\right]$ 而言，其在 (持续 $\Delta t \simeq 10^5 t_{\mathrm{Pl}}$ 时长的) 长时间动能主导时期取值为 $w(\phi) \simeq 1$，势能几乎保持不变。当 $t - t_B \simeq 10^5 t_{\mathrm{Pl}}$ 时，动能骤降且 $w(\phi) \to -1$，标志着势能占据主导后暴胀开始。因此，再加热前共有三个阶段: 反弹、动能转变，随后是暴胀 [146-148]。请注意，标准 LQC 中，反弹处暴胀子满足动能主导初条件时，也发现了类似行为 [211, 214]，且该结论对大范围的暴胀势都成立 [44, 183- 187]。

For a systematic study of mLQC-I, see [37, 38]. In addition, also within the minisuperspace approach of LQC, a reduced phase space quantization with an inflaton field and several different reference fields recovers many results of LQC, including showing that the resolution of the big bang singularity is robust [117].

有关 mLQC-I 的系统研究参见 [37, 38]。此外，同样在 LQC 的微超空间方法框架内，结合暴胀子与若干不同参考场的约化相空间量子化得到了许多与 LQC 一致的结果，还证明了大爆炸奇点的 resolving 是鲁棒的结论 [117]。

Cosmological perturbations have been studied in mLQC-I [8, 110, 118, 143, 149, 150], with the equations for the scalar and tensor perturbations the same as for standard LQC, with the only difference that in mLQC-I the effective background geometry (particularly the pre-bounce era) is different [146-148]. For the contracting phase, the background is well approximated by de Sitter contraction with $|aH| = -\eta^{-1}$, and the equations of the scalar and tensor perturbations reduce to those given in general relativity. Using the same arguments as in semi-classical cosmology, a natural choice for the initial state of perturbations in the contracting de Sitter space is the Bunch-Davies vacuum [72]

人们已经在 mLQC-I 中研究了宇宙学扰动 [8, 110, 118, 143, 149, 150]，其中标量扰动与张量扰动的方程和标准 LQC 一致，唯一区别在于 mLQC-I 中的有效背景几何 (尤其是反弹前阶段) 不同 [146-148]。对于收缩阶段，背景可以很好地用 $|aH| = -\eta^{-1}$ 对应的德西特收缩近似，标量与张量扰动方程可约化为广义相对论给出的形式。沿用半经典宇宙学中的论证，收缩德西特空间中扰动初态的自然选择是班奇-戴维斯真空 [72]

$$v_k^{(\text{ initial })} = \frac{1}{\sqrt{2k}} e^{-ik\eta} \left(1 - \frac{i}{k\eta}\right). \tag{21}$$

In classical slow-roll inflation, the background is also almost de Sitter, and at sufficient early times $a(\eta) \simeq 1/(-H\eta) \ll 1$ so $|\eta k| \simeq |H\eta| \gg 1$ and therefore the term $i/k\eta$ in (21) is negligible; as a result the modes (21) become indistinguishable from those defining the Minkowski vacuum $v_k^{(\text{Mink.})} = e^{-ik\eta}/\sqrt{2k}$. In contrast, for mLQC-I, far in the past contracting phase, the modes of interest lie outside the Hubble radius, so the term $i/k\eta$ in (21) cannot be neglected; for more details, see [150].

在经典慢滚暴胀中，背景同样近似为德西特空间，在足够早的时间 $a(\eta) \simeq 1/(-H\eta) \ll 1$ ，因此 $|\eta k| \simeq |H\eta| \gg 1$ ，故 (21) 式中的 $i/k\eta$ 项可忽略；结果 (21) 式的模式与定义闵氏真空 $v_k^{(\text{Mink.})} = e^{-ik\eta}/\sqrt{2k}$ 的模式不可区分。与之不同，对于 mLQC-I，在过去遥远的收缩阶段，我们关心的模式位于哈勃半径之外，因此不能忽略 (21) 式中的 $i/k\eta$ 项；更多细节参见 [150]。

The ambiguity in the form of the effective potential $\mathcal{U}$ appearing in the equation of motion of scalar perturbations (see the discussion below Eq. (12) on the origin of this ambiguity) remains in mLQC-I. However, contrary to standard LQC where different choices produce quite similar results for the power spectrum [11, 16], this is not the case for mLQC-I [143, 149]; in fact some choices have been already ruled out by current observations [149].

标量扰动运动方程中出现的有效势 $\mathcal{U}$ 的形式不确定性 (关于该不确定性的来源参见式 (12) 下方的讨论) 在 mLQC-I 中依然存在。但标准 LQC 中不同选择得到的功率谱结果十分相似 [11, 16], mLQC-I 并非如此 [143, 149]；实际上部分选择已经被现有观测排除 [149]。

Imposing Bunch-Davies initial conditions in the remote contracting phase and using the dressed metric approach, it was found that (similarly to standard LQC) the power spectra of the cosmological scalar and tensor perturbations can be divided into three regimes, ultraviolet, oscillatory, and infrared, respectively, corresponding to wavenumbers $k > k_{\text{LQC}}, k_{\text{I}} < k < k_{\text{LQC}}$, and $k < k_{\text{I}}$ (see Fig. 2 for the definition of k_{LQC} and k_I

) [8, 110, 149] . The major difference between the power spectra obtained in mLQC-I and standard LQC lies in the oscillatory and infrared regimes. First, in the oscillatory regime, the power spectrum of the scalar and tensor perturbations is proportional to k^{-3} in mLQC-I, as compared to k^{-1} in LQC [8]; however, this property does depend on the initial conditions of the scalar field and the choice of the potential [149]. Second, the scalar power spectrum was also studied in the hybrid approach [110, 118, 143], and, although the three different regimes mentioned above are also present in this case, some differences were found in the infrared and oscillatory regimes of mLQC-I, where a suppressed power spectrum was found for these infrared modes [143]. This is in striking contrast with the results of the dressed metric approach where the power spectrum is amplified for precisely these modes. Nevertheless, these differences arise only for the modes in the oscillatory and infrared regimes, while for the modes in the observable window (i.e., the ultraviolet regime), the differences are less than 1% [150]. Since the modes observed in the CMB today belong mainly to the ultraviolet regime, it may be difficult to distinguish LQC from mLQC-I observationally, as well as the dressed metric and hybrid quantization approaches that differ in mLQC-I. It is possible that other observables like non-Gaussianities may be able to distinguish these scenarios; this is a topic of current research.

在远收缩相施加邦奇-戴维斯初始条件并采用修正式量方法，研究发现 (与标准圈量子宇宙学类似) 宇宙学标量和张量扰动的功率谱可分为三个区域: 紫外区、振荡区和红外区，分别对应波数 $k > k_{\mathrm{LQC}}, k_{\mathrm{I}} < k < k_{\mathrm{LQC}}$ 和 $k < k_{\mathrm{I}}$ (k_{LQC} 与 k_I 的定义见图 2) [8, 110, 149] 。修正圈量子宇宙学 I(mLQC-I) 与标准圈量子宇宙学所得功率谱的主要差异存在于振荡区与红外区。首先，在振荡区内，mLQC-I 中标量与张量扰动的功率谱正比于 k^{-3} ，而圈量子宇宙学中正比于 k^{-1} [8]；但该性质依赖于标量场的初始条件和势的选取 [149]。其次，已有研究在混合方法框架下对标量功率谱展开分析 [110, 118, 143]，尽管上述三个不同区域在该情形下依然存在，但在 mLQC-I 的红外区和振荡区发现了一些差异: 红外模的功率谱受到抑制 [143]，这与修正式量方法的结果形成鲜明对比——后者恰好是这些模的功率谱被放大。尽管如此，这些差异仅出现在振荡区和红外区的模中，对于可观测窗口 (即紫外区) 的模，差异小于 1%[150]。由于当前宇宙微波背景观测到的模主要属于紫外区，因此可能很难在观测上区分圈量子宇宙学与 mLQC-I，也难以区分 mLQC-I 中不同的修正式量与混合量子化方法。非高斯性等其他可观测量或许能够区分这些情景，这是当前的研究课题。

Ekpyrosis in LQC

圈量子宇宙学中的火劫模型

Ekpyrosis is an alternative to inflation based on postulating the existence of a period of slow contraction due to the presence of an ultra-stiff fluid with $p > \rho$; for a review see [139]. Ekpyrosis is often taken to be a cyclic cosmology, with multiple recollapse and bounce cycles, although to generate perturbations that can match the observations of the CMB, only one collapse and bounce phase is necessary.

火劫模型是暴胀的替代方案，其假设存在一段由带有 $p > \rho$ 的超刚性流体驱动的慢收缩阶段；综述可见文献 [139]。火劫模型通常被归为循环宇宙学，包含多次坍缩与反弹循环，但若要生成符合宇宙微波背景观测结果的扰动，仅需一个坍缩反弹阶段。

Assuming there are multiple matter fields, a phase of ekpyrosis will generate nearly scale-invariant entropy perturbations (with a small red tilt), and these can in turn act as a source to excite density perturbations with the same nearly scale-invariant spectrum [96, 107, 138, 140] to match observations. On the other hand, tensor modes are never significantly excited during ekpyrosis [68], so a clear prediction for ekpyrotic cosmologies is a vanishingly small tensor-to-scalar ratio.

假设存在多个物质场，火劫阶段会生成近标度不变的熵扰动 (带有微小红倾斜)，这些熵扰动反过来可以作为源激发具有相同近标度不变谱的密度扰动 [96, 107, 138, 140]，从而匹配观测结果。另一方面，火劫过程中张量模从未被显著激发 [68]，因此火劫宇宙学的明确预言是张量标量比极小。

Although the ekpyrotic scenario was first proposed in string-inspired braneworld cosmology [137], the key ingredient to generate scale invariance-slow contraction due to an ultra-stiff fluid-is entirely independent of string theory. For example, in LQC it is possible to couple scalar fields with an appropriate potential to obtain the cyclic dynamics typically expected for ekpyrosis [83].

尽管火劫模型最初是在弦论启发的膜世界宇宙学中提出的 [137]，生成标度不变性的核心要素——超刚性流体带来的慢收缩——完全独立于弦论。例如，在圈量子宇宙学中，可以将标量场与合适的势耦合，得到火劫模型通常预期的循环动力学 [83]。

Another essential ingredient needed for any ekpyrotic model is a bounce, which can be provided by LQC. Further, since observationally relevant modes are super-horizon during the bounce, the equations of motion for the perturbations through the bounce can be taken from the separate universe quantization for long-wavelength modes [202, 205].

任何火劫模型都需要的另一核心要素是反弹，这一点可由圈量子宇宙学提供。此外，由于观测相关的模在反弹过程中处于超视界尺度，穿过反弹的扰动运动方程可以采用长波模的独立宇宙量子化得到 [202, 205]。

Solving the dynamics shows that the (nearly) scale-invariant curvature perturbations travel through the bounce unscathed, and they freeze once the background spacetime starts to expand after the bounce [203]. This demonstrates that LQC can complete the ekpyrotic paradigm, providing the bounce necessary to pass from a phase of slow contraction to our currently expanding universe. On the other hand, LQC does not modify the predictions in any way, so while ekpyrotic LQC is a viable cosmology, there are no predictions for LQC-specific effects in the CMB-the predictions for ekpyrosis are independent of the bounce mechanism.

求解动力学过程后可知，(近) 标度不变的曲率扰动可以完好无损地穿过反弹，且在反弹后背景时空开始膨胀时冻结 [203]。这证明圈量子宇宙学可以完善火劫范式，提供从慢收缩阶段过渡到我们当前膨胀宇宙所需的反弹。另一方面，圈量子宇宙学完全不会改变原有预言，因此尽管火劫圈量子宇宙学是自洽的宇宙学模型，宇宙微波背景中不存在圈量子宇宙学特有的预言——火劫模型的预言与反弹机制无关。

LQC Matter Bounce

LQC 物质反弹

Another alternative to inflation that relies on a cosmic bounce is the matter bounce scenario. In a contracting universe, if the dynamics are dominated by a matter (also often called dust) field with vanishing pressure, then cosmological perturbations become scale invariant as they exit the Hubble radius, assuming the modes were initially in the vacuum quantum state [198]. If there is a cosmic bounce, then these scale-invariant perturbations can provide appropriate initial conditions for the CMB [108]. A slight red tilt for the scalar power spectrum is obtained if the equation of state $w = p/\rho$ of the matter content is slightly negative [204], for example, due to a small contribution from dark energy [76,79].

依靠宇宙反弹的另一种暴胀替代方案是物质反弹场景。在收缩宇宙中，如果动力学由压强为零的物质 (也常称为尘埃) 场主导，那么假设模式初始处于真空量子态，宇宙学微扰在退出哈勃半径时会变得标度不变 [198]。如果发生宇宙反弹，这些标度不变的微扰就能为 CMB 提供合适的初始条件 [108]。如果物质组分的物态方程 $w = p/\rho$ 略为负值，标量功率谱就会得到轻微的红倾斜 [204]，例如这源于暗能量的微小贡献 [76,79]。

Since LQC automatically provides a bounce, with a matter-dominated era of contraction, it is one possible realization of the matter bounce scenario [204]. The simplest version of the LQC matter bounce is to assume a matter-dominated phase all the way to the bounce, but this scenario faces several difficulties. First, in this case the predicted amplitude of the scalar perturbations is determined by the energy density at the bounce (in natural units), so a Planckian bounce gives perturbations of order one and is clearly ruled out by observations (in addition to the fact that the regime of linear perturbation fails in this case). This problem can be avoided by requiring that the bounce occurs at scales far below the Planck scale, but then this seems somewhat unnatural from a quantum gravity perspective. Second, a matter-dominated contracting universe is unstable to the growth of anisotropies [142], which also suggests that a matter bounce scenario with vanishing pressure for the entire contracting phase requires extensive fine-tuning in the initial conditions for anisotropies to always remain small.

由于 LQC 能自然产生反弹，且拥有物质主导的收缩阶段，它是物质反弹场景的一种可能实现形式 [204]。最简版本的 LQC 物质反弹假设宇宙一直处于物质主导阶段直到反弹发生，但该场景面临若干困难。首先，这种情况下标量微扰的预言振幅由反弹时的能量密度 (自然单位制下) 决定，因此普朗克能标的反弹会产生量级为 1 的微扰，显然被观测排除 (此外该情况下线性微扰 regime 也不成立)。这个问题可以通过要求反弹发生在远低于普朗克能标的尺度来避免，但从量子引力的角度来看这略显不自然。其次，物质主导的收缩宇宙对各向异性的增长是不稳定的 [142]，这也说明整个收缩阶段压强都为零的物质反弹场景，需要对初始条件进行大量精细调节，才能让各向异性始终保持很小。

A scenario that alleviates these problems is to have the era of matter contraction followed by a period of ekpyrotic contraction before the LQC bounce [78, 144]. Then, the amplitude of the scalar perturbations is determined by the energy scale at the transition time between matter and ekpyrotic contraction (not the energy scale of the bounce) [77, 78, 174] , and the energy density of the ekpyrotic fluid grows more rapidly than anisotropies in a contracting universe, also alleviating the anisotropy problem [73, 75]. Even in this case, there are observational constraints from CMB data that provide very strong bounds on the possible strength of primordial anisotropies at the bounce [22], indicating that the anisotropy fine-tuning problem, although alleviated, is not entirely avoided by adding an ekpyrotic phase of contraction.

有一种场景可以缓解这些问题: 在 LQC 反弹前，物质收缩阶段之后会接一个火劫收缩阶段 [78, 144]。此时标量微扰的振幅由物质收缩转火劫收缩的过渡时刻的能标决定 (而非反弹本身的能标) [77, 78, 174]，并且在收缩宇宙中火劫流体的能量密度比各向异性增长更快，也能缓解各向异性问题 [73, 75]。即便如此，CMB 数据给出的观测约束已经对反弹处原初各向异性的可能强度给出了极强限制 [22]，结果表明虽然增加收缩的火劫阶段缓解了各向异性的精细调节问题，但并没有完全解决这个问题。

Another strong observational constraint on the matter bounce scenario is the latest bounds on the tensor-to-scalar ratio of $r < 0.036$ [4]. The simplest realizations of the matter bounce scenario typically predict a value for r close to 1, which is clearly ruled out. There are ways to suppress r , for example, by including several matter fields, setting the sound speed of the matter field to be small, or amplifying the scalar perturbations during the bounce, but these typically generate non-Gaussianities that violate observational bounds [74, 153, 175].

对物质反弹场景的另一项强观测约束来自最新对 $r < 0.036$ 张标比的限制 [4]。物质反弹场景最简单的实现形式通常预言 r 接近 1，这显然已经被排除。确实存在抑制 r 的方法，例如引入多个物质场、将物质场的声速设定为较小值，或是在反弹过程中放大标量微扰，但这些方法通常都会产生违反观测约束的非高斯性 [74, 153, 175]。

Interestingly, the LQC bounce can suppress the tensor-to-scalar ratio by decreasing the amplitude of tensor perturbations during the bounce [205], thereby alleviating this problem. The factor of suppression depends on the matter field at the bounce; for example, r decreases by a factor of 4 for radiation domination, and the suppression is stronger the closer the equation of state w is to 0 during the bounce. Note, however, that to significantly decrease r , it is necessary for the equation of state to be small, which reintroduces the anisotropy problem discussed above.

有趣的是，LQC 反弹可以在反弹过程中降低张量微扰的振幅，从而抑制张标比 [205]，进而缓解这个问题。抑制因子取决于反弹时的物质场；例如，辐射主导情况下 r 会降低为原来的 1/4，且反弹过程中物态方程 w 越接近 0，抑制效果越强。但需要注意，要显著降低 r 就要求物态方程足够小，这会重新引入上文讨论的各向异性问题。

In summary, LQC provides a simple realization of the matter bounce scenario, and LQC has the beneficial effect that it can successfully decrease the predicted tensor-to-scalar ratio. Despite this, it remains a challenge for the matter bounce scenario (whether realized in LQC or in some other bouncing cosmology) to simultaneously satisfy observational constraints from the CMB on the tensor-to-scalar ratio, non-Gaussianities, and primordial anisotropies.

综上，LQC 为物质反弹场景提供了一种简单的实现形式，并且 LQC 具备可以成功降低预言张标比的优势。尽管如此，对于物质反弹场景 (无论是 LQC 实现还是其他反弹宇宙学中的实现) 而言，同时满足 CMB 对张标比、非高斯性和原初各向异性的观测约束仍然是一项挑战。

Extensions

扩展

The discussion so far has focused on the simplest case: linear perturbations on an isotropic background. In this section, we discuss extensions to go beyond linear perturbation theory and allow for an anisotropic background spacetime.

到目前为止我们的讨论都集中在最简单的情况: 各向同性背景上的线性微扰。本节我们将讨论基础 LQC 微扰框架的扩展，研究超出线性微扰理论的情形，并允许背景时空存在各向异性。

Non-Gaussianity

非高斯性

The study of cosmological perturbations so far has been based on linear perturbation theory. Going to the next order in perturbation theory is important to demonstrate the viability of the theoretical framework by verifying that the perturbation theory remains under control and the compatibility of its predictions with observations. This is a non-trivial demand, since self-interactions between perturbations are mediated by effective couplings of gravitational origin, and these couplings could become large when curvature invariants reach the Planck scale. This strategy has also been followed for many scenarios for the early universe such as inflation [155], ekpyrosis [141], and the matter bounce [74].

迄今为止，宇宙学扰动的研究一直基于线性扰动理论。将扰动理论推进到下一阶至关重要: 这可以验证扰动理论仍处于可控范围，也可检验其预言与观测的相容性，从而证明理论框架的合理性。这一要求并非易事，因为扰动之间的自相互作用由引力起源的有效耦合传递，当曲率不变量达到普朗克尺度时, 这些耦合可能会变得很大。该研究策略也已应用于早期宇宙的诸多场景, 例如暴胀 [155]、火劫宇宙 [141] 和物质反弹 [74]。

It is, therefore, of interest to compute the corrections that self-interactions between perturbations introduce in cosmological observables. If these corrections are large, then the perturbation expansion fails. But even if the perturbation theory is shown to be under control, interactions among perturbations can still be strong enough to generate sizable non-Gaussian correlations in the CMB, while there are strong observational upper bounds [3].

因此，计算扰动间的自相互作用给宇宙学可观测量带来的修正是很有意义的。如果这些修正很大，扰动展开就会失效。但即便证明了扰动理论处于可控范围，扰动之间的相互作用仍可能足够强，在 CMB 中产生可观的非高斯关联，而目前观测已经给出了严格的上限 [3]。

A detailed analysis of non-Gaussianity within LQC has been carried out by extending the dressed metric approach to next-to-leading order in perturbations [12]. See [7, 208, 215] for preliminary work in this direction, where part of these non-Gaussianities were discussed.

通过将修饰度规方法扩展到微扰的次领头阶，对圈量子宇宙学 (LQC) 中的非高斯性进行了详细分析 [12]。有关此方向的初步工作，请参阅 [7, 208, 215] ，其中讨论了部分非高斯性。

There are two quantities of interest: corrections to the power spectrum $\Delta P_{\mathcal{R}}(k)$ induced by next-to-leading order contributions as well as the size and form of the three-point correlation function $\langle \mathcal{R}_{\mathbf{k}_1} \mathcal{R}_{\mathbf{k}_2} \mathcal{R}_{\mathbf{k}_3} \rangle$

that is identically zero in the linear (or Gaussian) approximation. We start by reviewing results for the three-point function.

有两个感兴趣的量: 次领头阶贡献对功率谱 $\Delta P_{\mathcal{R}}(k)$ 的修正，以及在线性 (或高斯) 近似下恒为零的三点关联函数 $\langle \mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3}\rangle$ 的大小和形式。我们首先回顾三点函数的结果。

The three-point functions are quantified by the bispectrum $B_{\mathcal{R}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ defined by $\langle \mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3}\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_{\mathcal{R}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$. The Dirac delta distribution is a consequence of homogeneity and enforces that $\mathbf{k}_1,\mathbf{k}_2$, and $\mathbf{k}_3$ form a triangle. It is convenient to encode the amplitude of the bispectrum in a dimensionless function $f_{\mathrm{NL}}(\mathbf{k}_1,\mathbf{k}_2)$, defined as

三点函数由 $B_{\mathcal{R}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ 定义的双谱量化，双谱由 $\langle \mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3}\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_{\mathcal{R}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ 给出。狄拉克 δ 分布是均匀性的结果，它约束 $\mathbf{k}_1,\mathbf{k}_2$ 和 $\mathbf{k}_3$ 构成三角形。将双谱的振幅编码到无量纲函数 $f_{\mathrm{NL}}(\mathbf{k}_1,\mathbf{k}_2)$ 中会更方便，该函数定义为

$$B_{\mathcal{R}}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = f_{\mathrm{NL}}(\mathbf{k}_1,\mathbf{k}_2)\left[\frac{4\pi^4}{k_1^3k_2^3}P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + \text{ cyclic permutations}\right], \tag{22}$$

where $P_{\mathcal{R}}(k_i)$ is the power spectrum and the last two terms are obtained from the first one by cyclic permutations of k_1,k_2 , and k_3 . The $f_{\mathrm{NL}}(\mathbf{k}_1,\mathbf{k}_2)$ has been computed numerically in LQC with a post-bounce inflationary phase [12] (assuming a vacuum state for the perturbations far in the contracting branch when all modes of interest were deep inside the curvature radius), with the following results.

其中 $P_{\mathcal{R}}(k_i)$ 是功率谱，最后两项由第一项通过 k_1,k_2 和 k_3 的循环置换得到。$f_{\mathrm{NL}}(\mathbf{k}_1,\mathbf{k}_2)$ 已在圈量子宇宙学中对反弹后的暴胀阶段进行了数值计算 [12](假设所有感兴趣模式都远在曲率半径内部时，收缩分支深处的扰动处于真空态)，得到以下结果。

First, $f_{\mathrm{NL}}(\mathbf{k}_1,\mathbf{k}_2)$ is highly oscillatory around a small number of the order of the slow-roll parameters (and equal to the value of f_{NL} in standard inflation); see Fig. 2 in [12]. Similar oscillations also appear for the power spectrum, although in that case the oscillations do not average to a small number (see Fig. 2); these oscillations originate from the oscillatory nature of scalar perturbations.

首先，$f_{\mathrm{NL}}(\mathbf{k}_1,\mathbf{k}_2)$ 在慢滚参数量级的小数值附近高度振荡 (与标准暴胀中 f_{NL} 的值相等)；参见文献 [12] 的图 2。功率谱也会出现类似的振荡，不过在这种情况下，振荡的平均结果不是小数 (参见图 2)；这些振荡源自标量扰动的振荡特性。

For large wavenumbers $k \gtrsim k_{\mathrm{LQC}}, f_{\mathrm{NL}}(\mathbf{k}_1,\mathbf{k}_2)$ reduces to the well-known prediction from slow-roll inflation that $|f_{\mathrm{NL}}| \sim 10^{-2}$ [155], as expected given the discussion at the end of the subsection "Inflation in the Dressed Metric and Hybrid Approaches"-this is a good test of the calculations. On the other hand, when the three wavenumbers k_1,k_2 , and k_3 are smaller than k_{LQC} , then $|f_{\mathrm{NL}}|$ grows to $\sim 10^3$. The form of f_{NL} is peaked on wavenumber configurations for which $k_3 \ll k_2 \approx k_1$ and $k_3+k_2 \approx k_1$; these are called, respectively, squeezed and flattened configurations.

对于大波数，$k \gtrsim k_{\mathrm{LQC}}, f_{\mathrm{NL}}(\mathbf{k}_1, \mathbf{k}_2)$ 退化为慢滚暴胀中广为人知的预言 $|f_{\mathrm{NL}}| \sim 10^{-2}$ [155]，这与“修正式量度中的暴胀与混合方法”小节末尾的讨论结论一致，是对计算结果的良好检验。另一方面，当三个波数 k_1, k_2 和 k_3 都小于 k_{LQC} 时，$|f_{\mathrm{NL}}|$ 会增长到 $\sim 10^3$ 。f_{NL} 的形式在满足 $k_3 \ll k_2 \approx k_1$ 和 $k_3 + k_2 \approx k_1$ 的波数组态上出现峰值；这两种组态分别被称为压缩组态和平坦组态。

For $k > k_I$ (where k_I is defined as the most infrared mode that exits the horizon during inflation; see Fig. 2), the modulus of $f_{\mathrm{NL}}(\mathbf{k}_1, \mathbf{k}_2)$ can be approximated by [12]

对于 $k > k_I$ (其中 k_I 被定义为暴胀期间出视界的最红外模式，参见图 2)，$f_{\mathrm{NL}}(\mathbf{k}_1, \mathbf{k}_2)$ 的模可近似为 [12]

$$|f_{\mathrm{NL}}(\mathbf{k}_1, \mathbf{k}_2)| \simeq F_{\mathrm{NL}} e^{-\alpha(k_1+k_2+k_3)/k_{LQC}}, \ \alpha = \sqrt{\frac{\pi}{12}} \cdot \frac{\Gamma[5/6]}{\Gamma[4/3]} \approx 0.647, \tag{23}$$

where the amplitude F_{NL} is found numerically to be $\sim 10^3$. This shows that $f_{\mathrm{NL}}(\mathbf{k}_1, \mathbf{k}_2)$ is scale dependent in LQC, with the dependence dictated by k_{LQC} ; this is a distinctive feature due to the LQC bounce. Like for the power spectrum, LQC effects for non-Gaussianities only become important for wavenumbers comparable to (or smaller than) k_{LQC} , whose value depends on the number of e -folds of inflation. Note that Eq. (23) is an approximation for the modulus, as it does not include the oscillations in $f_{\mathrm{NL}}(\mathbf{k}_1, \mathbf{k}_2)$; an improved approximation which does include the oscillations has been recently introduced [132]. Also, for $k < k_I$ the bispectrum quickly falls off, as also happens for the power spectrum, as can be seen in Fig. 2; hence k_I can be seen as an infrared cutoff.

其中振幅 F_{NL} 经数值计算得 $\sim 10^3$ 。这表明在圈量子宇宙学 (LQC) 中 $f_{\mathrm{NL}}(\mathbf{k}_1, \mathbf{k}_2)$ 具有标度依赖性，依赖关系由 k_{LQC} 决定；这是 LQC 反弹带来的独特特征。与功率谱的情况类似，仅当波数与 k_{LQC} 相当 (或小于 k_{LQC}) 时，LQC 对非高斯性的效应才会变得显著，其数值取决于暴胀的 e 折叠数。请注意，式 (23) 是模的近似结果，因为它未包含 $f_{\mathrm{NL}}(\mathbf{k}_1, \mathbf{k}_2)$ 中的振荡；最近已有研究提出了包含振荡的改进近似 [132]。此外，与功率谱的情况一致，当处于 $k < k_I$ 时，双谱会快速衰减，这可从图 2 中看出；因此 k_I 可被视为红外截断。

The predicted amplitude $|f_{\mathrm{NL}}(\mathbf{k}_1, \mathbf{k}_2)|$ is smaller than the upper bounds on non-Gaussianities obtained from CMB observations [3]. Furthermore, the recent analysis in [132] shows that even if k_{LQC} lies in the observational window of the CMB, the oscillations of $f_{\mathrm{NL}}(\mathbf{k}_1, \mathbf{k}_2)$ are very effective in washing away the imprint in the CMB bispectrum of the non-Gaussianity produced at the bounce. (This is because the CMB bispectrum is obtained from $f_{\mathrm{NL}}(\mathbf{k}_1, \mathbf{k}_2)$ by smearing out $\mathbf{k}_1$ and $\mathbf{k}_2$ against spherical Bessel functions, and the oscillations drastically reduce the result of this smearing.) Hence, it is not expected that such non-Gaussianities will be found in the CBM bispectrum. This is compatible with the recent data analysis performed in [95, 195]. However, this does not mean that the primordial perturbations are Gaussian in inflationary models in LQC, but instead it implies that we need to look elsewhere to find its effects. One such possibility could be the large-scale anomalies in the CMB power spectrum, as discussed in the subsection ”Inflation in the Dressed Metric and Hybrid Approaches”.

预测的振幅 $|f_{\mathrm{NL}}(\mathbf{k}_1,\mathbf{k}_2)|$ 小于 CMB 观测得到的非高斯性上限 [3]。此外，文献 [132] 的最新分析表明，即使 k_{LQC} 处于 CMB 的观测窗口内，$f_{\mathrm{NL}}(\mathbf{k}_1,\mathbf{k}_2)$ 的振荡也会非常有效地抹去反弹产生的非高斯性在 CMB 三重谱中的印记。(这是因为 CMB 三重谱是通过将 $f_{\mathrm{NL}}(\mathbf{k}_1,\mathbf{k}_2)$ 中的 $\mathbf{k}_1$ 和 $\mathbf{k}_2$ 对球贝塞尔函数做积分抹除得到的，振荡会大幅降低该抹除过程的结果。) 因此，我们并不预期这类非高斯性会在 CMB 三重谱中被发现，这与文献 [95, 195] 的最新数据分析结果一致。但这并不意味着 LQC 中暴胀模型的原初扰动是高斯型的，反而说明我们需要在其他方向寻找它的效应。其中一种可能是 CMB 功率谱中的大尺度反常，正如小节“修正面量暴胀与混合方法”中所讨论的那样。

Finally, the correction to the power spectrum $\Delta P_{\mathcal{R}}(k)$ at next-to-leading order in perturbations has been shown to be $\Delta P_{\mathcal{R}}/P_{\mathcal{R}} \sim \varepsilon f_{\mathrm{NL}} P_{\mathcal{R}}$ [12], where $\varepsilon \lesssim 10^{-2}$ is the slow-roll parameter during inflation. This expression produces $\Delta P_{\mathcal{R}}/P_{\mathcal{R}} \lesssim 10^{-4}$. Therefore, although the amplitude f_{NL} in LQC is larger by several orders of magnitude than its value in slow-roll inflation, the perturbative expansion remains valid. The smallness of $\Delta P_{\mathcal{R}}/P_{\mathcal{R}}$ in LQC, even though $|f_{\mathrm{NL}}|$ is large, is due to the fact that higher-order perturbative corrections are proportional to higher powers of $P_{\mathcal{R}}(k)$, and $P_{\mathcal{R}}(k) \lesssim 10^{-7}$ is small. In this sense, $P_{\mathcal{R}}(k)$ is the small ”parameter” that ensures the validity of the perturbative expansion.

最后，研究表明扰动次领头阶对功率谱 $\Delta P_{\mathcal{R}}(k)$ 的修正为 $\Delta P_{\mathcal{R}}/P_{\mathcal{R}} \sim \varepsilon f_{\mathrm{NL}} P_{\mathcal{R}}$ [12]，其中 $\varepsilon \lesssim 10^{-2}$ 是暴胀期间的慢滚参数。该表达式给出 $\Delta P_{\mathcal{R}}/P_{\mathcal{R}} \lesssim 10^{-4}$ 。因此，尽管 LQC 中振幅 f_{NL} 比慢滚暴胀中的对应值大几个数量级，微扰展开仍然有效。即使 $|f_{\mathrm{NL}}|$ 很大，LQC 中 $\Delta P_{\mathcal{R}}/P_{\mathcal{R}}$ 仍然很小，这是因为高阶微扰修正正比于 $P_{\mathcal{R}}(k)$ 的更高次幂，而 $P_{\mathcal{R}}(k) \lesssim 10^{-7}$ 很小。从这个意义上来说，$P_{\mathcal{R}}(k)$ 正是保证微扰展开有效性的小“参数”。

Anisotropies

各向异性

Anisotropies are an important topic in bouncing cosmologies, since they rapidly grow in a contracting universe. In general relativity (and in absence of sources of anisotropy), in homogeneous spacetimes the shears are proportional to the inverse cube of the scale factor, so even the tiniest deviation from isotropy would tend to grow rapidly in the contracting phase of the cosmos, more rapidly than contributions from cold dark matter and radiation fluids, and could dominate the dynamics. It is therefore important to check how the predictions for the CMB summarized above (and derived assuming isotropy) may change if some degree of anisotropy is included. This has been analyzed in detail for the simplest anisotropic (though still homogeneous) background spacetime, namely, the Bianchi I geometries [19,20,22].

各向异性是反弹宇宙学中的一个重要课题，因为它会在收缩宇宙中迅速增长。在广义相对论中 (且不存在各向异性源时)，均匀时空内的切变与标度因子的三次方成反比，因此即使宇宙收缩阶段各向同性存在极微小偏离，也会迅速增长，增长速度超过冷暗物质和辐射流体的贡献，甚至可能主导动力学。因此有必要检验上文总结的 (假设各向同性推导得到的)CMB 预测，在包含一定程度各向异性后会如何变化。这一点已经针对最简单的各向异性 (但仍均匀) 背景时空，即比安基 I 几何做了详细分析 [19,20,22]。

For the homogeneous background, a study of inflation in anisotropic Bianchi I models within the effective theory of LQC shows that, first, the attractor character of inflation persists and, second, solutions of the effective equation which are of phenomenological interest, the universe isotropizes both to the future and past of the bounce [123]. This shows that the so-called cosmic no-hair theorem of general relativity [196] that anisotropies in the early universe are generically washed out by the expansion remains true in LQC.

对于均匀背景，圈量子宇宙学有效理论框架下对各向异性比安基 I 模型的暴胀研究表明: 第一，暴胀的吸引子特性得以保留；第二，对于有唯象学意义的有效方程解，宇宙在反弹的未来和过去都会各向同性化 [123]。这说明广义相对论中“早期宇宙的各向异性通常会被膨胀冲刷掉”的所谓宇宙无毛定理，在圈量子宇宙学中仍然成立。

On the other hand, cosmological perturbations retain memory of the anisotropies for much longer than the background geometry does [19, 20, 22]. This is because anisotropic features in the perturbations do not dilute as the universe expands, although they are red-shifted in the sense that wavenumbers $\mathbf{k}$ of the modes with non-zero anisotropies can be red-shifted to super-Hubble scales and made inaccessible to observations. Since the red-shift is linear with the scale factor, unless the accumulated expansion in the cosmic history is much larger than what it is commonly accepted, perturbations can evade the cosmic no-hair theorem and imprint some degree of anisotropy in the CMB, even though anisotropies in the background geometry may be completely diluted by the expansion.

另一方面，宇宙扰动保留各向异性记忆的时间比背景几何长得多 [19, 20, 22]。这是因为扰动中的各向异性特征不会随宇宙膨胀稀释，只是它们会发生红移: 带有非零各向异性的模式，其波数 $\mathbf{k}$ 可以红移到哈勃视界外，变得无法被观测。由于红移与标度因子呈线性关系，除非宇宙历史中的累积膨胀远大于通常接受的数值，否则即使背景几何中的各向异性已经被膨胀完全稀释，扰动仍能避开宇宙无毛定理，在 CMB 中留下一定程度各向异性的印记。

The theory of cosmological perturbations on a background Bianchi I geometries is significantly more complicated than its counterpart on FLRW spacetimes. The equations of motion for scalar and tensor perturbations can be derived for classical general relativity by expanding Einstein’s equations [172], as well as in a Hamiltonian treatment, better adapted to LQC, where the quantization can also be completed [18, 21]. Anisotropies introduce two main new features for the dynamics of the perturbations, by modifying the effective potentials in the perturbative equations of motion such that they (i) now depend on the direction of the wavenumber $\mathbf{k}$ and (ii) couple scalar and tensor modes. These potentials induce anisotropies in the perturbations, even if their initial state is isotropic, as well as correlations—and quantum entanglement—among scalar and tensor perturbations. In particular, the scalar and tensor power spectra of the isotropic theory are replaced by a family of direction-dependent power spectra $P_{ss'}(\mathbf{k})$, with $s, s' = 0, +2, -2$, where $s = \pm 2$ describe circularly polarized tensor modes. Anisotropies in $P_{ss'}(\mathbf{k})$ can be quantified by their angular multipoles, obtained by expanding the spectra in spin-weighted spherical harmonics

比安基 I 背景几何上的宇宙扰动理论，远比 FLRW 时空上的对应理论复杂。标量和张量扰动的运动方程可以通过对爱因斯坦方程做展开得到经典广义相对论下的形式，也可以在更适配圈量子宇宙学的哈密顿框架下处理，并且能完成量子化 [18, 21]。各向异性通过修改扰动运动方程中的有效势，给扰动动力学带来两个主要新特性:(i) 有效势现在依赖于波数 $\mathbf{k}$ 的方向，(ii) 标量模式和张量模式相互耦合。这些势会诱发扰动中的各向异性，即使初始态是各向同性的，还会带来标量扰动和张量扰动之间的关联以及量子纠缠。具体来说，各向同性理论中的标量和张量功率谱被一族方向依赖的功率谱 $P_{ss'}(\mathbf{k})$ 取代，满足 $s, s' = 0, +2, -2$，其中 $s = \pm 2$ 描述圆偏振张量模式。$P_{ss'}(\mathbf{k})$ 中的各向异性可以通过角多极矩量化，角多极矩由谱按自旋加权球谐函数展开得到

$$P_{ss'}(\mathbf{k}) = \sum_{L=|s-s'|}^{\infty} \sum_{M=-L}^{L} P_{ss'}^{LM}(k)\, {}_{s-s'}Y_{LM}(\hat{k}), \tag{24}$$

where ${}_{s-s'}Y_{LM}(\hat{k})$ is a spherical harmonic of weight $s-s'$. The use of spin-weighted spherical harmonics guarantees that all $P_{ss'}^{LM}(k)$ are scalars under rotations. The coefficients $P_{ss'}^{LM}(k)$, which depend only on the modulus of $\mathbf{k}$, encode the information about anisotropies in the primordial perturbations and are all zero in the isotropic limit except for $L = M = 0$. In Bianchi I geometries, L is constrained to take even values, if the initial state for perturbations is parity invariant. Multipolar components $P_{ss'}^{LM}(k)$ with $s \neq s'$ describe correlations between different perturbations.

其中 ${}_{s-s'}Y_{LM}(\hat{k})$ 是权重为 $s-s'$ 的球谐函数。使用旋加权球谐函数可保证所有 $P_{ss'}^{LM}(k)$ 在旋转下均为标量。仅依赖于 $\mathbf{k}$ 模长的系数 $P_{ss'}^{LM}(k)$ 编码了原初扰动的各向异性信息，在各向同性极限下，除 $L = M = 0$ 外所有系数均为零。在毕安基 I 型几何中，若扰动的初态具有宇称不变性，则 L 被约束为偶数。带 $s \neq s'$ 的多极分量 $P_{ss'}^{LM}(k)$ 描述不同扰动之间的关联。

The functions $P_{ss'}^{LM}(k)$ have been computed in LQC for inflationary models [19, 20] as well as for the ekpyrotic and matter bounce scenarios [22], with the common prediction in all these models that anisotropies in the primordial spectra are dominated by the quadrupolar contribution $L = 2$. The key difference between different models is the predicted scale dependence of the quadrupole: for inflationary models, $P_{ss'}^{2M}(k)$ scales approximately as $1/k$, while for ekpyrosis and the matter bounce, $P_{ss'}^{2M}(k)$ is almost scale invariant. A quadrupolar modulation in the CMB is the hallmark of primordial anisotropies, and its scale dependence can be used to distinguish between inflation and some of its alternatives.

在圈量子宇宙学 LQC 中，人们已经针对暴胀模型 [19,20] 以及火劫和物质反弹场景 [22] 计算得到了函数 $P_{ss'}^{LM}(k)$，所有这些模型的共同预言是，原初谱中的各向异性由四极贡献 $L = 2$ 主导。不同模型的核心区别在于对四极标度依赖性的预言: 对于暴胀模型，$P_{ss'}^{2M}(k)$ 近似按 $1/k$ 标度，而对于火劫模型和物质反弹，$P_{ss'}^{2M}(k)$ 几乎是标度不变的。CMB 中的四极调制是原初各向异性的标志，其标度依赖性可用于区分暴胀和部分替代模型。

Interestingly, a quadrupolar modulation has been detected in the CMB [24], although its statistical significance is low, since the chances that such quadrupole could have been generated in an isotropic universe as a result of a statistical fluke are high. Nonetheless, the observed quadrupole may have a natural explanation in primordial anisotropies generated by a cosmic bounce [19, 20, 22]. Future observations, particularly if tensor modes are detected, will help to clarify the origin of the quadrupole and its scale dependence.

有趣的是，人们已经在 CMB 中探测到四极调制 [24]，不过它的统计显著性很低，因为这种四极是统计涨落在各向同性宇宙中产生的概率很高。尽管如此，观测到的四极可以用宇宙反弹产生的原初各向异性给出自然的解释 [19,20,22]。未来观测，尤其是如果探测到张量模态，将帮助厘清四极及其标度依赖性的起源。

It is also possible to compute angular correlation functions in the CMB, $C_{\ell\ell'}^{XX'}$ where $X, X' = T, E, B$ refers to temperature, electric, and magnetic polarization in the CMB, including their cross-correlations, in LQC [19, 22]. In particular TB and EB correlations are forbidden by symmetry arguments in FLRW spacetimes, while they are non-zero in Bianchi I and are therefore a smoking gun for anisotropies. Finally, Ref. [19] also contains a Markov chain Monte Carlo analysis (MCMC), using TT, EE, TE, and lensing data to find the best fit to the six free cosmological parameters $\Omega_b, \Omega_c, \theta_{MC}, \tau, A_s$, and n_s in presence of anisotropies.

在 LQC 中也可以计算 CMB 中的角关联函数 $C_{\ell\ell'}^{XX'}$，其中 $X, X' = T, E, B$ 指 CMB 中的温度、电偏振和磁偏振，包括它们的互关联 [19, 22]。特别地，FLRW 时空的对称性禁止 TB 和 EB 关联存在，而这二者在毕安基 I 型几何中不为零，因此是各向异性的确凿证据。最后，参考文献 [19] 还包含了马尔可夫链蒙特卡洛分析 (MCMC)，该分析使用 TT、EE、TE 和引力透镜数据，在存在各向异性的情况下得到了六个自由宇宙学参数 $\Omega_b, \Omega_c, \theta_{MC}, \tau, A_s$ 和 n_s 的最优拟合结果。

Limitations

局限性

LQC has been able to provide a detailed possible picture for the Planck era of the universe. This scenario has been applied to extend existing viable cosmological models based on general relativity to include Planck scale physics. The inclusion of cosmological perturbations described in this chapter has led to a better understanding of the way Planck scale physics could be imprinted in the observables we have access to at present, mainly through the CMB.

圈量子宇宙学 (LQC) 已经能够为宇宙的普朗克时代提供一幅详细的可能图景。这一框架已被用于推广基于广义相对论的现有可行宇宙学模型，将普朗克尺度物理纳入其中。本章所描述的宇宙微扰的引入，让人们更好地理解了普朗克尺度物理如何能主要通过宇宙微波背景 (CMB) 留在我们如今可观测的信号中。

The main limitations of this research program are rooted in the absence of a complete theory of quantum gravity. This is what motivated the development of LQC at first, as a symmetry reduced version of LQG. These limitations are carried over to the description of perturbations. The strategy followed so far for LQC and cosmological perturbations is a common one in physics: start from the simplest scenario, and add complications in a sequential manner, in order to test the robustness of the predictions. One expects that, although fine details may change as one builds more complete models, the main features will be robust. This strategy has been very successful in the history of physics, from the study of the hydrogen atom to black holes. Regarding perturbations in LQC, this was the motivation to study anisotropies and non-Gaussianity. But there still remain several avenues where the robustness of the theoretical framework needs to be further tested. We summarize in this section the status of the main two limitations, namely, the absence of a loop quantization for cosmological perturbations and the existence of quantization ambiguities.

该研究方向的主要局限性根源在于缺乏一套完整的量子引力理论，这正是 LQC 最初作为圈量子引力 (LQG) 对称约化版本被开发出来的动因。这些局限性也延续到了微扰描述中。迄今为止，LQC 和宇宙微扰采用的研究策略是物理学中的常用思路: 从最简单的情景出发，逐步逐步增加复杂度，以此检验预言的稳健性。学界普遍认为，尽管构建更完整模型时细节可能发生变化，但核心特征会保持稳健。这一策略在物理学史上从氢原子研究到黑洞研究都非常成功，研究 LQC 中的各向异性和非高斯性也正是出于这一考量。但目前仍有多个方向需要进一步检验理论框架的稳健性。我们在本节总结了两大局限性的研究现状，即宇宙微扰尚不存在圈量子化方案，以及量子化歧义的存在。

Trans-Planckian Modes

跨普朗克模式

In LQG, the simplest geometric observable is the two-dimensional area of a surface. The spectrum of the area operator is discrete and has the important property that there is a minimum non-zero area eigenvalue $\Delta \sim \ell_{\mathrm{Pl}}^2$, called the area gap. In this sense, there is an ultraviolet cutoff for the area in LQG. This raises the obvious question: is there an ultraviolet cutoff for the wavelength of cosmological perturbations? In other words, do there exist cosmological perturbations with a trans-Planckian wavelength $\lambda < \ell_{\mathrm{Pl}}$, or not? This is the trans-Planckian problem in cosmology.

在圈量子引力 (LQG) 中，最简单的几何可观测量是曲面的二维面积。面积算符的谱是离散的，且具有一个重要性质: 存在最小非零面积本征值 $\Delta \sim \ell_{\mathrm{Pl}}^2$ ，称为面积间隙。从这个意义上说，LQG 中的面积存在一个紫外截断。这就引出了一个显而易见的问题: 宇宙学扰动的波长是否也存在紫外截断? 换句话说，是否存在波长为跨普朗克尺度的宇宙学扰动 $\lambda < \ell_{\mathrm{Pl}}$？这就是宇宙学中的跨普朗克问题。

Note that the trans-Planckian problem is of particular interest for inflationary models, since if the inflationary period is sufficiently long then some of the CMB modes would have been trans-Planckian at the onset of inflation, in which case the trans-Planckian problem becomes observationally relevant. But even for alternatives to inflation like ekpyrosis or the matter bounce, the trans-Planckian problem is still present as a conceptual problem even though it may not be observationally relevant.

需要注意的是，跨普朗克问题在暴胀模型中尤其值得关注，因为如果暴胀期足够长，那么部分宇宙微波背景 (CMB) 模式在暴胀开始时就已经是跨普朗克模式，这种情况下跨普朗克问题就与观测相关。但即使对于火劫论或物质反弹这类暴胀替代模型，跨普朗克问题虽然可能与观测无关，它仍然是一个存在的概念性问题。

The possibility of an ultraviolet cutoff for cosmological perturbations, motivated by quantum gravity, has been proposed in various contexts [130, 136, 157, 200]. On the one hand, it could cure ultraviolet divergences in the quantum theory, but on the other hand, it seems to require a dynamical number of degrees of freedom since the ultraviolet cutoff is $k < 1/a(t)\,\ell_{\mathrm{Pl}}$ (although this would presumably not be a problem for full LQG where the Hilbert space includes all possible graphs). A naïve cutoff can also induce unwanted violations of local Lorentz invariance: even if the cutoff is at the Planck scale, such a violation can produce prohibitively large effects at low energy due to the integrals in k appearing in radiative corrections [86].

由量子引力引出的宇宙学扰动存在紫外截断的可能性, 已经在多个研究背景中被提出 [130, 136, 157, 200]。一方面, 这可以解决量子理论中的紫外发散问题, 但另一方面, 它似乎要求自由度数量是动力学的，因为紫外截断是 $k < 1/a(t)\,\ell_{\mathrm{Pl}}$ (尽管这对完整 LQG 来说大概不成问题，完整 LQG 的希尔伯特空间包含了所有可能的图)。朴素的截断还会引发不必要的局域洛伦兹不变性破坏: 即使截断处于普朗克尺度，由于辐射修正中 k 出现的积分，这种破坏也会在低能区产生大到无法接受的效应 [86]。

While the presence of an ultraviolet cutoff for the LQG area observable is suggestive that there may also be a cutoff for the wavelengths of cosmological perturbations in LQC, it is not conclusive. There are several proposals for length operators in LQG [45, 154, 191] , and while they have a discrete spectrum, numerics suggest eigenvalues can be arbitrarily close to 0 [70]. This discussion is quite preliminary, and more work is needed to determine whether there is or not an ultraviolet cutoff for the length operator and the implications for cosmological perturbations.

LQG 面积可观测量存在紫外截断这一点暗示 LQC 中的宇宙学扰动波长可能也存在截断，但这并不是定论。LQG 中存在多个关于长度算符的提案 [45, 154, 191]，这些长度算符的谱都是离散的，但数值计算表明其本征值可以任意接近 0[70]。目前相关讨论还十分初步，需要更多研究来确定长度算符是否存在紫外截断，以及这对宇宙学扰动有何影响。

In the dressed metric approach, the adiabatically renormalized energy and pressure densities of the perturbations always remain below the Planck scale, even near the bounce [11]. Since LQC effects only become important for the background dynamics when the energy density approaches the Planck scale, this (combined with the absence of an obvious minimum length in LQG) suggests that it may be sufficient to concentrate on LQC effects on the background degrees of freedom and to assume that LQC effects in perturbations are subleading, in particular for the longest wavelengths we observe in the CMB. This motivates the Fock quantization of perturbations in both the dressed metric and hybrid approaches. But it is a strong assumption, and it is desirable to reinforce it from the point of view of full LQG.

在 dress 度规方法中，经绝热重整的扰动能量密度和压强密度始终保持在普朗克尺度以下，即使在反弹附近也是如此 [11]。由于只有当能量密度接近普朗克尺度时，LQC 效应才会对背景动力学产生重要影响，这一点 (结合 LQG 中不存在明显最小长度的结论) 说明，我们或许只需要研究 LQC 对背景自由度的效应即可，假设 LQC 对扰动的效应是次要的，对于我们在 CMB 中观测到的最长波长来说尤其如此。这就是 dress 度规方法和混合方法中对扰动进行福克量子化的动机。但这是一个很强的假设，从完整 LQG 的角度出发，有必要对这个假设进一步验证。

For a phenomenological study of potential trans-Planckian effects in LQC, see [162] where modified dispersion relations motivated by analog gravity [193] were considered; the result is that if there are sufficiently many e -folds of inflation, in some cases these modified dispersion relations can have an impact on predictions for the CMB. It remains to be seen whether modified dispersion relations can arise due to LQC effects on cosmological perturbations and if so what specific modified dispersion relation captures the LQC corrections.

关于 LQC 中跨普朗克效应的唯象研究，可参考文献 [162]，该工作研究了由类比引力引出的修正色散关系 [193]；结论是，如果暴胀存在足够多的 e 暴胀折叠，在某些情况下这些修正色散关系会对 CMB 的预言产生影响。修正色散关系是否真的可以由 LQC 对宇宙学扰动的效应产生，如果可以，又是哪一种具体的修正色散关系能够描述 LQC 修正，这些问题都仍有待研究。

It would be nice to study this question using a loop quantization of perturbations, like the approach based on the separate universe framework [202] summarized above, but this approach is limited to long-wavelength modes, and consequently it cannot shed any insight on the trans-Planckian problem; see the subsection ”Separate Universe Loop Quantization” for details. Instead, it may be necessary to go beyond LQC and study cosmology from full LQG; for recent work in this direction, see the section ”Beyond LQC”.

我们也可以通过扰动的圈量子化来研究这个问题，比如上文总结的基于独立宇宙框架的方法 [202]，但该方法仅限于长波长模式，因此无法为跨普朗克问题提供任何见解；详见“独立宇宙圈量子化”小节。相反，我们或许有必要跳出 LQC 的范围，从完整 LQG 出发研究宇宙学；相关最新研究可参见“超越圈量子宇宙学”章节。

In summary, whether the possibility of a Planckian cutoff is realized or not in LQC, it is clearly important to understand the impact of LQG effects on trans-Planckian modes: are they ruled out by LQG, or do they exist with possibly modified dynamics? More generally, recall that for FLRW spacetimes, LQC effects (as expressed for sharply peaked states that can be well approximated by an effective description) modify the classical equations of motion with terms of the order $\rho/\rho_{\rm Pl}$, and it is well understood that corrections of this type affect cosmological perturbations as well. For Fourier modes of perturbations whose wavelength nears (or perhaps is even shorter than) $\ell_{\rm Pl}$, it is possible that LQG effects may modify their dynamics with extra quantum corrections of the form $\lambda/\ell_{\rm Pl}$, with the exact form of these corrections remaining to be determined. These are important open questions that could lead to a much better understanding of quantum gravity effects in cosmology.

总而言之，无论圈量子宇宙学 (LQC) 中是否存在普朗克截断，理解圈量子引力 (LQG) 效应对跨普朗克模式的影响显然十分重要: 跨普朗克模式是被 LQG 排除，还是会以经过修正的动力学形式存在？更广义而言，需要注意，对于 FLRW 时空，LQC 效应 (体现在可以用有效描述很好近似的尖峰态中) 对经典运动方程的修正项量级为 $\rho/\rho_{\rm Pl}$ ，且目前已经明确，这类修正也会对宇宙学扰动产生影响。对于波长接近 (甚至可能短于) $\ell_{\rm Pl}$ 的扰动傅里叶模式，LQG 效应可能会通过 $\lambda/\ell_{\rm Pl}$ 形式的额外量子修正改变其动力学，而这类修正的确切形式仍有待确定。这些都是重要的开放性问题，解决它们能够极大增进我们对宇宙学中量子引力效应的理解。

Quantization Ambiguities

量子化歧义

A second problem in LQC (as well as in LQG or, more generally, in non-linear quantum theories) is the presence of ambiguities that arise in the quantization process. Generally speaking, there are two types of quantization ambiguities that are relevant for cosmological perturbations: ambiguities arising in the loop quantization of the FLRW background and ambiguities concerning the perturbations themselves.

LQC(以及 LQG，更广泛地说所有非线性量子理论) 的第二个问题是，量子化过程会产生歧义。一般而言，与宇宙学微扰相关的量子化歧义分为两类:FLRW 背景圈量子化过程中产生的歧义，以及和微扰本身相关的歧义。

There are several sources of quantization ambiguities for the loop quantization of the FLRW spacetime, and these ambiguities will of course affect the dynamics of perturbations evolving on this background. These include ambiguities in the definition of the curvature and inverse volume operators [188], as well as the ambiguities related to modified LQC that have been discussed above in the subsection ”A First Look at Quantum Ambiguities: Inflation in Modified LQC”, specifically which form of the Hamiltonian constraint to quantize and whether it is necessary to use Thiemann’s identity to express the extrinsic curvature as an operator. The first category of ambiguities is less important as it leads to some quantitative differences in the quantum evolution, but qualitatively the dynamics are not significantly affected by these ambiguities, at least for the flat FLRW spacetime. On the other hand, the ambiguities of modified LQC produces important differences in the quantum dynamics of the background spacetime in the contracting branch before the bounce and can have an impact on the perturbations evolving on this background, as explained in the subsection ”A First Look at Quantum Ambiguities: Inflation in Modified LQC”. For this reason, it is important to better understand how to address these ambiguities properly.

FLRW 时空的圈量子化存在多个导致量子化歧义的来源，这些歧义当然会影响在该背景下演化的微扰的动力学。这些歧义包括曲率和逆体积算符定义中的歧义 [188]，以及在“初探量子歧义: 修正 LQC 中的暴胀”小节中讨论过的与修正 LQC 相关的歧义，具体来说就是要对哪种形式的哈密顿约束进行量子化，以及是否有必要使用蒂曼恒等式将外曲率表示为算符。第一类歧义不太重要，因为它只会导致量子演化出现一些定量差异，但至少对于平坦的 FLRW 时空而言，这些歧义在定性上不会显著影响动力学。另一方面，正如“初探量子歧义: 修正 LQC 中的暴胀”小节所解释的，修正 LQC 的歧义会在反弹前的收缩分支中导致背景时空的量子动力学出现重要差异，并可能对在该背景下演化的微扰产生影响。因此，更好地理解如何妥善处理这些歧义非常重要。

Another ambiguity is closely related to the problem of time. In LQC, the problem of time is usually addressed by using a matter field as a relational clock, so that the quantum evolution of the cosmological wave function is calculated with respect to the matter field. While this is a natural way to address the problem of time in this context, in general there may be multiple matter fields present, and then there is an ambiguity: which matter field should be used as a relational clock? Further, it is also possible to use geometric clocks as well as matter clocks [116, 158], which clearly increases the number of possible clocks. Presumably, it will always be possible to choose different clocks, but (although this point is clear in the classical theory) relating the quantum evolution with respect to different relational clocks is not always simple [113, 194].

另一个歧义与时间问题密切相关。在圈量子宇宙学 (LQC) 中，通常通过使用物质场作为相对时钟来解决时间问题，这样宇宙波函数的量子演化就相对于该物质场进行计算。虽然在这种情况下这是解决时间问题的自然方式，但一般来说可能存在多个物质场，于是就产生了一个歧义: 应该使用哪个物质场作为相对时钟？此外，除了物质时钟，也可以使用几何时钟 [116, 158]，这显然增加了可能的时钟数量。据推测，总是可以选择不同的时钟，但 (尽管这一点在经典理论中很明确) 将相对于不同相对时钟的量子演化联系起来并不总是那么简单 [113, 194]。

Finally, the last ambiguity is due to the absence of a complete loop quantization for cosmological perturbations. There are four main approaches that have been developed, as reviewed above: the dressed metric approach, hybrid quantization, the separate universe framework for LQC, and effective dynamics. All of these approaches have their strengths, although none can be considered complete. At this time, it is necessary to make a choice on which approach to use (although the dressed metric and hybrid approaches give very similar results [145]). As discussed at the beginning of this section, an important open problem is to understand how to perform a loop quantization for all cosmological perturbations (and which will hopefully give these four approaches in various limits). Note that clarifying the connection to full LQG may help both with resolving these ambiguities and developing a loop quantization for the perturbations.

最后一处模糊性源于宇宙学微扰还没有完整的圈量子化方案。正如前文综述，目前已发展出四种主要方法: dressed metric 方法、混合量子化、LQC 的独立宇宙框架，以及有效动力学。所有这些方法都有各自的优势，但没有一种是完备的。目前必须对所用方法做出选择 (尽管 dressed metric 方法和混合方法给出的结果非常相似 [145])。正如本节开头所述，一个重要的开放性问题是，如何对所有宇宙学微扰进行圈量子化 (有望在不同极限下退化为上述四种方法)。需要注意的是，厘清其与完整 LQG 的联系，既可能帮助解决这些模糊性，也可能推动微扰圈量子化方案的发展。

Beyond LQC

超出圈量子宇宙学 (LQC) 的研究

LQC is based upon applying the quantization tools of LQG to cosmological spacetimes. But LQC is a symmetry-reduced model, in which the symmetries of the spacetimes of interest are imposed at the classical level, before quantization. The processes of quantization and symmetry reduction do not commute in general, so there is no guarantee that LQC does not miss some important aspects of LQG. Symmetry reduction has been useful to make progress from black holes to atomic physics, but it is clearly desirable to eventually connect LQC with full LQG. Further, since there are ambiguities in the definition of LQG itself, connecting LQC to LQG may also give some suggestions on how to address the ambiguities of LQG. There have been some promising results-for both background and perturbative degrees of freedom-coming from three different directions: canonical LQG, covariant spin foam models, and group field theory.

LQC 基于将圈量子引力 (LQG) 的量子化工具应用于宇宙学时空中。但 LQC 是对称性约化模型，在量子化之前的经典层面就施加了目标时空的对称性。量子化过程和对称性约化过程通常不对易，因此无法保证 LQC 不会遗漏 LQG 的某些重要方面。对称性约化从黑洞研究到原子物理都对推进研究有所助益，但显然最终仍需要将 LQC 与完整 LQG 建立联系。此外，由于 LQG 本身的定义存在歧义，将 LQC 与 LQG 建立联系也能为解决 LQG 的歧义提供思路。目前已经从三个不同方向得到了一些很有前景的结果——涵盖背景自由度和微扰自由度，这三个方向分别是正则 LQG、协变自旋泡沫模型和群场论。

For the homogeneous and isotropic FLRW spacetime, there has been significant progress in extracting cosmology from full LQG. To do this requires several nontrivial steps. The first step is to pick a certain approach to LQG and a particular definition of the dynamics; given the quantization ambiguities in LQG, it is interesting to explore different possibilities and determine whether some choices are preferred over others. The second step is to choose a certain class of quantum states that may correspond to cosmological spacetimes; this is done by determining how to appropriately impose the conditions of homogeneity and isotropy on quantum states (note that these conditions cannot be imposed exactly due to quantum fluctuations). The third step is conceptually simple but often technically difficult, which is to evaluate the quantum dynamics, as prescribed by step 1, on the states chosen in step 2. In general, approximations are often needed in this step since the quantum dynamics typically is not tangential to the subspace of cosmological states chosen in step 2.

针对均匀各向同性 FLRW 时空，从完整 LQG 推导出宇宙学已经取得了重大进展。这一过程需要多个非平凡步骤。第一步是选择特定的 LQG 研究方法和动力学定义；考虑到 LQG 中存在量子化歧义，探索不同可能性并判断部分选择是否更优是很有意义的。第二步是选择一类可能对应宇宙学时空中的量子态；这需要确定如何对量子态恰当地施加均匀性和各向同性条件 (注意由于量子涨落，这些条件无法精确施加)。第三步概念上简单但通常技术难度很高: 它是在第二步选出的量子态上，计算第一步规定的量子动力学。一般来说，这一步往往需要近似，因为量子动力学通常不与第二步选出的宇宙态子空间相切。

For canonical LQG, there has been work relating LQC to full LQG at both the kinematical and dynamical levels. Kinematically, it has been shown how to define a diffeomorphism-invariant notion of a homogeneous and isotropic sector of LQG and how to embed the LQC kinematical Hilbert space into the one of LQG

[27, 41, 42, 71, 101 − 104, 109] (see [43] for an extension to Bianchi I geometries). Multiple approaches have been developed at the dynamical level; three based on studying coherent states on a fixed graph structure in LQG are as follows: (i) the quantum-reduced loop gravity approach where the additional symmetry that the metric be diagonal is imposed on the quantum theory as a weak constraint, reducing the SU(2) labels on the graph to U(1) quantum numbers [25,26]; (ii) using complexifier coherent states and treating the Euclidean and Lorentzian terms in the scalar constraint separately as in full LQG, which is related to the modified LQC theory mLQC-I (see the subsection "A First Look at Quantum Ambiguities: Inflation in Modified LQC") [90,91]; and (iii) using a path-integral reformulation of LQG [126] as well as a perturbative expansion of the Hamiltonian constraint [210]; these three models also give results similar in some ways to LQC or modified LQC, although with some important drawbacks concerning the classical limit [92, 171] and not matching some aspects of the LQC/mLQC-I dynamics [33, 209] known in the LQC literature as the "improved dynamics." To address these shortcomings, the above work has since been extended to allow for graph-changing dynamics for the approach based on the path-integral reformulation of LQG [128], while other approaches have also been developed with a key ingredient being length-dependent holonomies [48, 127]. Both of these approaches, with graph-changing or length-dependent holonomies, give the correct classical limit. Further, the dynamics in the Planck regime are very similar to mLQC-I for the graph-changing LQG dynamics [128], while the approach based on length-dependent holonomies gives dynamics very similar to standard LQC [48, 127].

对于正则 LQG，已有研究在运动学和动力学层面都建立了 LQC 与完整 LQG 的联系。在运动学层面，已经证明如何定义微分同胚不变的 LQG 均匀各向同性区概念，以及如何将 LQC 运动学希尔伯特空间嵌入 LQG 的希尔伯特空间 [27, 41, 42, 71, 101 − 104, 109] (对 Bianchi I 几何的推广参见文献 [43])。动力学层面已经发展出多种方法，其中三种基于研究 LQG 中固定图结构上的相干态，分别是:(i) 量子约化圈引力方法: 该方法在量子理论中将度规对角化作为弱约束施加额外对称性，将图上的 SU(2) 标签约化为 U(1) 量子数 [25,26]；(ii) 像完整 LQG 中一样，使用复相干态，分别处理标量约束中的欧几里得项和洛伦兹项，对应修正 LQC 理论 mLQC-I(参见小节“量子歧义初探: 修正 LQC 中的暴胀”)[90,91]；(iii) 使用 LQG 的路径积分重表述 [126] 以及哈密顿约束的微扰展开 [210]；这三种模型在某些方面也得到了与 LQC 或修正 LQC 相似的结果，不过在经典极限方面存在重要缺陷 [92, 171]，并且与 LQC 文献中称为“改进动力学”的 LQC/mLQC-I 动力学的部分内容不匹配 [33, 209]。为了解决这些不足，上述基于 LQG 路径积分重表述的研究后续已经被扩展，支持改变图结构的动力学 [128]，同时也发展出了其他方法，其核心要素是依赖长度的和乐 [48, 127]。这两种支持变图或依赖长度和乐的方法都能得到正确的经典极限。此外，变图 LQG 动力学在普朗克能标的动力学与 mLQC-I 非常相似 [128]，而基于依赖长度和乐的方法得到的动力学与标准 LQC 非常相似 [48, 127]。

For the covariant spin foam approach, the dynamics is based on the spin foam vertex amplitude for vertices of arbitrary valence defined in [134], while the quantum states are coarse triangulations of a 3-sphere, capturing large-scale degrees of freedom [46,47,176]. It seems likely necessary to extend these models to allow for either graph-changing or length-dependent holonomies, as in the canonical case, to recover the correct dynamics. But already some important qualitative similarities to LQC arise, like a non-singular bounce, and suggest the connection between LQG and LQC will be possible in spin foam models as well.

对于协变自旋泡沫方法，动力学基于文献 [134] 中定义的任意价顶点的自旋泡沫顶点振幅，而量子态是三维球的粗三角化, 刻画了大尺度自由度 [46,47,176]。和正则情形一样, 要得到正确动力学, 似乎必须扩展这些模型以容纳变图或长度依赖和乐。但它已经展现出了和圈量子宇宙学一些重要的定性相似性，比如非奇异反弹，这表明在自旋泡沫模型中也可以建立 LQG 与 LQC 的关联。

For group field theory (GFT), the dynamics is based on the quantum equations of motion for the GFT field, and the quantum states corresponding to FLRW spacetimes are assumed to be condensate states, where homogeneity is imposed by assuming every quantum of geometry is in the state [115]; this approach is based on viewing cosmology as the hydrodynamical limit of GFT [169]. Using a massless scalar field as a relational clock, the dynamics can be extracted in a relational form, giving dynamics very similar (and in fact identical for a specific class of states) to the LQC dynamics in the limit that the non-linear term in the dynamics is negligible [170]. For further work on the cosmological sector of GFT, see [94, 112, 173, 207].

对于群场论 (GFT)，动力学基于 GFT 场的量子运动方程，对应 FLRW 时空的量子态被认为是凝聚态，通过假设每个几何量子都处于相同态来实现均匀性 [115]；该方法将宇宙学视为 GFT 的流体动力学极限 [169]。以无质量标量场作为关系时钟，可以提取出关系形式的动力学，在动力学中非线性项可忽略的极限下，所得动力学与 LQC 动力学非常相似 (对特定一类态甚至完全相同)[170]。关于 GFT 宇宙学 sector 的更多研究，参见 [94, 112, 173, 207]。

There has also been work in studying cosmological perturbations starting from full LQG. There was some early work based on the quantum-reduced loop gravity approach [125, 168], and importantly this has since been extended to use length-dependent holonomies [128]. Also, quantum correlations between different spatial regions have been studied in spin foam models [120], and two approaches to perturbations in GFT have been studied, one based on using matter fields as reference rods to localize perturbations [114, 156] and the other on an extension of the separate universe approach for long-wavelength scalar perturbations to GFT states [111].

目前也有研究从完整 LQG 出发研究宇宙学微扰。早期有基于约化量子圈引力方法的工作 [125, 168]，重要的是，该方向此后已被扩展到使用长度依赖和乐 [128]。此外，已有研究在自旋泡沫模型中探讨不同空间区域之间的量子关联 [120]；GFT 中也研究了两种微扰方法，一种以物质场为参考杆定位微扰 [114, 156]，另一种是将长波长标量微扰的分离宇宙方法扩展到 GFT 态 [111]。

The progress made so far in understanding the relation between LQC and LQG is encouraging, but remains incomplete. The three approaches developed so far each have their strengths and weaknesses. Canonical LQG offers a relatively direct path to recover cosmological dynamics, but it is difficult to handle graph-changing dynamics which have been argued to be important. It may be possible to sidestep this problem by using length-dependent holonomies, but this is a new ingredient that would also need to be implemented in full LQG. The work in the spin foam approach suggests using especially simple quantum states to describe cosmological spacetimes, but it seems that more complicated states (i.e., more refined triangulations of a 3-sphere) will be needed for cosmological perturbation theory, especially to describe short-wavelength perturbations. And for GFT, condensate states seem to capture the notion of homogeneous states extremely well, with their dynamics very similar to LQC, but these condensate states ignore the graph structure of the quantum state, and this may lead to a too-rapid growth in the strength of the nonlinear term in the GFT dynamics.

目前在理解 LQC 与 LQG 关系上取得的进展令人鼓舞，但仍不完备。已发展的三种方法各有优劣：正则 LQG 提供了一条相对直接的路径得到宇宙学动力学，但很难处理被认为十分重要的变图动力学；通过使用长度依赖和乐或许可以绕过这个问题，但这是一个新要素，还需要在完整 LQG 中实现。自旋泡沫方法的研究表明，可用特别简单的量子态描述宇宙学时空，但宇宙学微扰论，尤其是描述短波长微扰，似乎需要更复杂的态 (即三维球更精细的三角化)。而对于 GFT，凝聚态对均匀态的刻画非常出色，其动力学也与 LQC 非常相似，但这类凝聚态忽略了量子态的图结构，这可能导致 GFT 动力学中非线性项强度增长过快。

It is our hope that future work in these various directions will address these challenges and further improve our understanding of quantum gravity effects in cosmology, both at the background and perturbative levels.

我们希望，未来这些不同方向的研究能够解决这些挑战，进一步加深我们对宇宙学中量子引力效应的理解，无论是背景层面还是微扰层面。

Cross-References

交叉引用

Loop Quantum Cosmology: Physics of Singularity Resolution and Its Implications

循环量子宇宙学: 奇点消解的物理学及其启示

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